Chapter 3. Gravitation and Mechanics

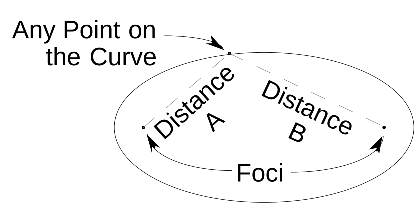
Objectives: Upon completion of this chapter you will be able to describe the force of gravity, characteristics of ellipses, and the concepts of Newton's principles of mechanics. You will be able to recognize acceleration in orbit and explain Kepler's laws in general terms. You will be able to describe tidal effect and how it is important in planetary systems.

Gravitation is the mutual attraction of all masses in the universe. The concepts associated with planetary motions developed by Johannes Kepler (1571-1630) describe the positions and motions of objects in our solar system. Isaac Newton (1643-1727) explained why Kepler's laws worked, in terms of gravitation. Since planetary motions are orbits, and all orbits are ellipses, a review of ellipses follows.

Ellipses

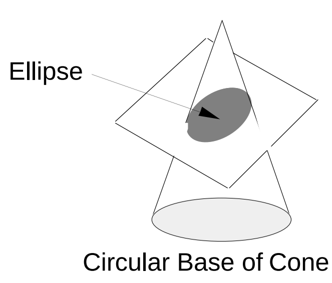
An ellipse is a closed plane curve generated in such a way that the sums of its distances from two fixed points (called the foci) is constant. In the illustration below, Distance A + B is constant for any point on the curve.

Ellipse Foci

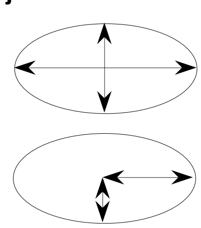


An ellipse also results from the intersection of a circular cone and a plane cutting completely through the cone. The maximum diameter is called the major axis. It determines the size of an ellipse. Half the maximum diameter, the distance from the center of the ellipse to one end, is called the semi-major axis.

An Ellipse from a Conical Section

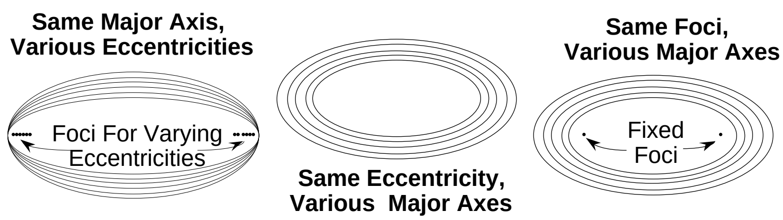


Major And Minor Axes



Semi-major and Semi-minor Axes

The shape of an ellipse is determined by how close together the foci are in relation to the major axis. Eccentricity equals the distance between the foci divided by the major axis. If the foci coincide, the ellipse is a circle. Therefore, a circle is an ellipse with an eccentricity of zero.



Recap

1. The definition of an ellipse is a closed plane curve generated in such a way that the sums of its distances from the two fixed points (the foci) is ___________________ .
2. Eccentricity equals the distance between the foci divided by the __________________ axis.
3. If the foci coincide, the ellipse is a ____________________ .

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1. constant   2. major   3. circle

Newton’s Principles of Mechanics

Newton realized that the force that makes apples fall to the ground is the same force that makes the planets “fall” around the sun. Newton had been asked to address the question of why planets move as they do. He established that a force of attraction toward the sun becomes weaker in proportion to the square of the distance from the sun.

Newton postulated that the shape of an orbit should be an ellipse. Circular orbits are merely a special case of an ellipse where the foci are coincident. Newton described his work in the *Mathematical Principles of Natural Philosophy* (often called simply the *Principia*), which he published in 1685. Newton gave his laws of motion as follows:

I. Every body continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.

II. The change of motion (linear momentum) is proportional to the force impressed and is made in the direction of the straight line in which that force is impressed.

III. To every action there is always an equal and opposite reaction; or, the mutual actions of two bodies upon each other are always equal, and act in opposite directions.

(Notice that Newton’s laws describe the behavior of inertia, they do not explain its nature.)

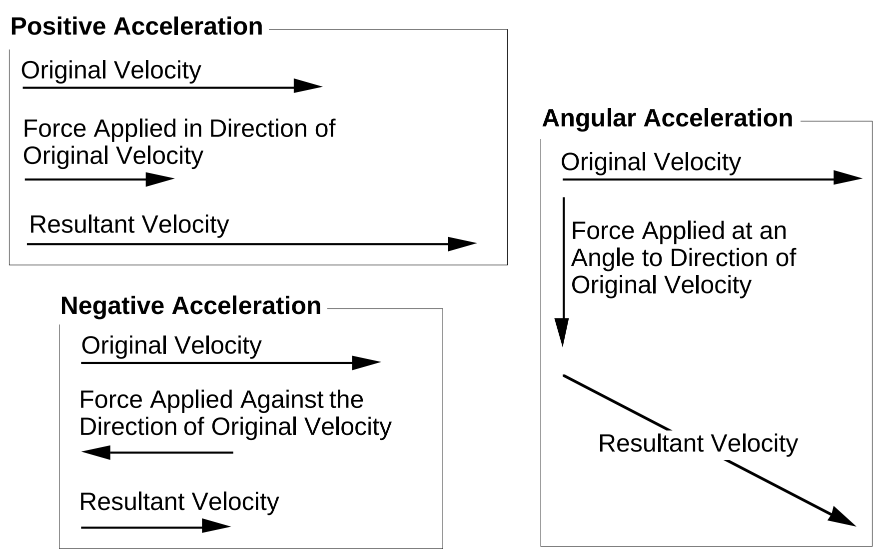
There are three ways to modify the momentum of a body. The mass can be changed, the velocity can be changed (acceleration), or both.

mass (M) x change in velocity (acceleration, A) = force (F)

or

F = MA

Acceleration may be produced by applying a force to an object. If applied in the same direction as an object's velocity, the velocity increases in relation to an unaccelerated observer. If acceleration is produced by applying a force in the opposite direction from the object's original velocity, it will slow down. If the acceleration is produced by a force at some other angle to the velocity, the object will be deflected.



In order to measure mass, we must agree to a standard. The world standard of mass is the kilogram, whose definition is based on the mass of a metal cylinder kept in France. Previously, the standard was based upon the mass of one cubic centimeter of water being one gram (this is approximately correct). Force can now be expressed numerically. A unit of force is the dyne, which is equal to the force required to accelerate a mass of 1 gram 1 cm per second per second. Another is the Newton, the force required to accelerate a 1-kg mass 1 m/sec^2. A Newton is equal to the weight of about 100 grams of water, or about 1/2 cup.

Recap

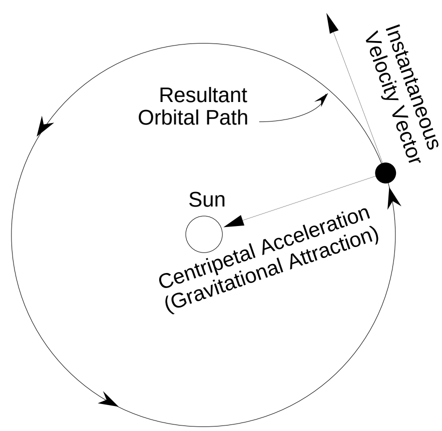
1. Gravitation is the mutual ___________________ of all masses in the universe.
2. Newton postulated that the shape of an orbit should be an ________________.
3. Circular orbits are merely a special case of an ellipse where the foci are _______________________ .
4. If acceleration is produced in the opposite direction of velocity, the object will ____________________ _______________ .

1. attraction   2. ellipse   3. coincident   4. slow down

Acceleration in Orbit

Newton's first law describes how, once in motion, planets remain in motion. What it does not do is explain how the planets are observed to move in nearly circular orbits rather than straight lines. Enter the second law. To move in a curved path, a planet must have an acceleration toward the center of the circle. This is called centripetal acceleration and is supplied by the mutual gravitational attraction between the sun and the planet.

Motion in a Circular Orbit



Kepler's Laws

Kepler's laws, as expressed by Newton, are:

I. If two bodies interact gravitationally, each will describe an orbit that is a conic section about the common mass of the pair. If the bodies are permanently associated, their orbits will be ellipses. If they are not permanently associated with each other, their orbits will be hyperbolas (open curves).

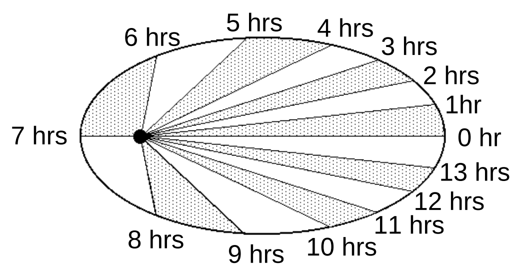
III If two bodies revolve around each other under the influence of a central force (whether or not in a closed elliptical orbit), a line joining them sweeps out equal areas in the orbital plane in equal intervals of time.

III. If two bodies revolve mutually about each other, the sum of their masses times the square of their period of mutual revolution is in proportion to the cube of the semi-major axis of the relative orbit of one about the other.

The major application of Kepler's first law is to precisely describe the geometric shape of an orbit: an ellipse, unless perturbed by other objects. Kepler's first law also informs us that if a comet, or other object, is observed to have a hyperbolic path, it will visit the sun only once, unless its encounter with a planet alters its trajectory again.

Kepler's second law addresses the velocity of an object in orbit. Conforming to this law, a comet with a highly elliptical orbit has a velocity at closest approach to the sun that is many times its velocity when farthest from the sun. Even so, the area of the orbital plane swept is still constant for any given period of time.

Time Versus Area Swept by an Orbit



Kepler's third law describes the relationship between the masses of two objects mutually revolving around each other and the determination of orbital parameters. Consider a small star in orbit about a more massive one. Both stars actually revolve about a common center of mass, which is called the barycenter. This is true no matter what the size or mass of each of the objects involved. Measuring a star's motion about its barycenter with a massive planet is one method that has been used to discover planetary systems associated with distant stars.

Obviously, these statements apply to a two-dimensional picture of planetary motion, which is all that is needed for describing orbits. A three-dimensional picture of motion would include the path of the sun through space.

Gravity Gradients (Tidal Forces)

Gravity's strength is inversely proportional to the square of the objects' distance from each other. For an object in orbit about a planet, the parts of the object closer to the planet feel a slightly stronger gravitational attraction than do parts on the other side of the object. This is known as gravity gradient. It causes a slight torque to be applied to any mass which is non-spherical and non-symmetrical in orbit, until it assumes a stable attitude with the more massive parts pointing toward the planet. An object whose mass is distributed like a bowling pin would end up in an attitude with its more massive end pointing toward the planet, if all other forces were equal.

In the case of a fairly massive body such as our moon in Earth orbit, the gravity gradient effect has caused the moon, whose mass is unevenly distributed, to assume a stable rotational rate, which keeps one face towards Earth at all times, like the bowling pin described above.

The moon acts upon the Earth's oceans and atmosphere, causing two bulges to form. The bulge on the side of the Earth that faces the moon is caused by the proximity of the moon and its relatively stronger gravitational pull on that side. The bulge on the opposite side of the earth results from that side being attracted toward the moon less strongly than is the central part of the earth. The earth's crust is also affected to a small degree. Other factors, including Earth's rotation and surface roughness, complicate the tidal effect. On planets or satellites without oceans, the same forces apply, but they cause slight deformations in the body rather than oceanic tides. This mechanical stress can translate into heat as in the case of Jupiter's volcanic Io.

Recap

1. To move in a circular path a planet must have applied a constant acceleration toward the center of the circle. This acceleration is called ________________ acceleration.
2. When closest to the sun, an object is moving at higher velocity than when it is farthest from the sun; however, the __________ of the orbital plane swept is constant for any given period of time.
3. (Two) stars actually revolve about a common center of mass, which is called the _________________ .
4. For an object in orbit about a planet, the parts of the object closer to the planet feel a slightly stronger... attraction than do parts on the other side of the object. This is known as __________________ __________________ .

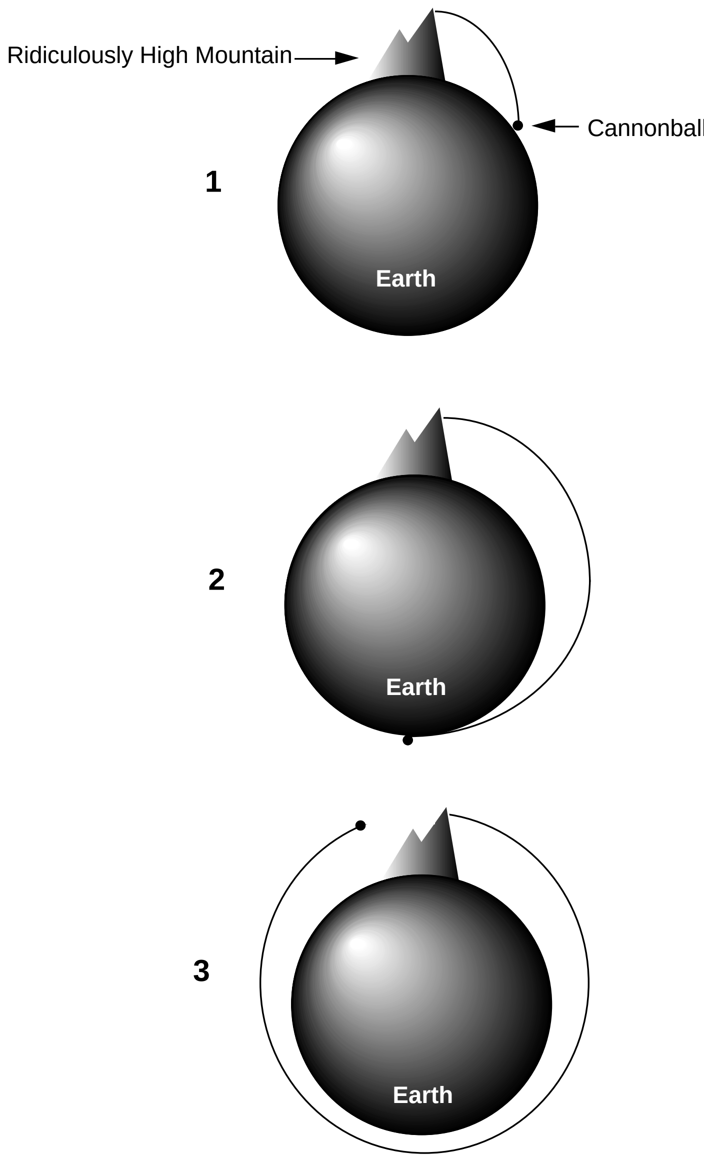
1. centripetal 2. area 3. barycenter 4. gravity gradient

How Orbits Work

The drawings on the next page simplify the physics of orbiting Earth. We see Earth with a huge, tall mountain rising from it. The mountain, as Isaac Newton first envisioned, has a cannon at its summit. When the cannon is fired, the cannonball follows its ballistic arc, falling as a result of Earth's gravity, and it hits Earth some distance away from the mountain. If we put more gunpowder in the cannon, the next time it's fired, the cannonball goes halfway around the planet before it hits the ground. With still more gunpowder, the cannonball goes so far that it just never touches down at all. It falls completely around Earth. It has achieved orbit.

If you were riding along with the cannonball, you would feel as if you were falling. The condition is called free fall. You'd find yourself falling at the same rate as the cannonball, which would appear to be floating there (falling) beside you. You'd just never hit the ground. Notice that the cannonball has not escaped Earth's gravity, which is very much present—it is causing the mass to fall. It just happens to be balanced out by the speed provided by the cannon.

In the third drawing in the figure, you'll see that part of the orbit comes closer to Earth's surface that the rest of it does. This is called the periapsis of the orbit. It also has various other names, depending on which body is being orbited. For example, it is called perigee at Earth, perijove at Jupiter, periselene or perilune in lunar orbit, and perihelion if you're orbiting the sun. In the drawing, the mountain represents the highest point in the orbit. That's called apoapsis (apogee, apojove, aposelene, apolune, aphelion). The time it takes, called the orbit period, depends on altitude. At space shuttle altitudes, say 200 kilometers, it's 90 minutes.



The cannonball provides us with a pretty good analogy. It makes it clear that to get a spacecraft into orbit, you need to raise it up (the mountain) to a high enough altitude so that Earth's atmosphere isn't going to slow it down too much. You have to accelerate it until it is going so fast that as it falls, it just falls completely around the planet.

In practical terms, you don't generally want to be less than about 150 kilometers above the surface of Earth. At that altitude, the atmosphere is so thin that it doesn't present much frictional drag to slow you down. You need your rocket (or cannon) to speed the spacecraft up to the neighborhood of 30,000 kilometers (about 19,000 miles) per hour. Once you've done that, your spacecraft will continue falling around Earth. No more propulsion is necessary, except for occasional minor adjustments. These very same mechanical concepts apply whether you're talking about orbiting Earth, the moon, the sun, or anything. Only the terms and numbers are different. The cannonball analogy is good, too, for talking about changes you can make to an orbit. Looking at the third drawing, imagine that the cannon has still more gunpowder in it, sending the cannonball out a little faster. With this extra speed, the cannonball will miss Earth's surface by a greater margin. The periapsis altitude is raised by increasing the spacecraft's speed at apoapsis.

This concept is very basic to space flight. Similarly, decrease the speed when you're at apoapsis, and you'll lower the periapsis altitude. Likewise, if you increase speed when you're at periapsis, this will cause the apoapsis altitude to increase. Decelerating at periapsis will lower the apoapsis.

Recap

1. Periapsis is also called _______________ at Jupiter and _______________ orbiting the sun.
2. The periapsis altitude is ________________ by increasing speed at apoapsis.
3. Decelerating at periapsis will _______________ the apoapsis altitude.

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1. perijove . . . perihelion   2. raised   3. lower

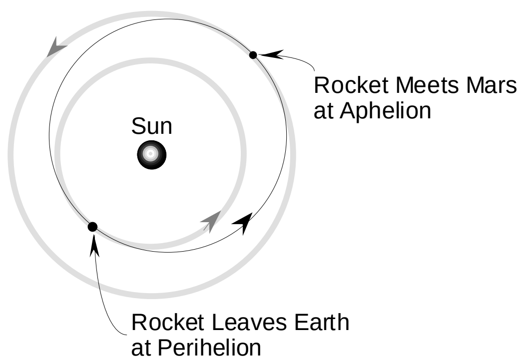
Chapter 4. Interplanetary Trajectories

Objectives: Upon completion of this chapter you will be able to describe the use of Hohmann transfer orbits in general terms, and how spacecraft use them for interplanetary travel. You will be able to describe in general terms the exchange of angular momentum between planets and spacecraft on gravity assist trajectories.

Hohmann Transfer Orbits

To launch a spacecraft to an outer planet such as Mars, using the least propellant possible, first consider that the spacecraft is already in solar orbit as it sits on the launch pad. Its existing solar orbit must be adjusted to cause it to take the spacecraft to Mars. In other words, the spacecraft's perihelion (closest approach to the sun) will be Earth's orbit, and the aphelion (farthest distance from the sun) will intercept the orbit of Mars at a single point. This is called a Hohmann Transfer Orbit. The portion of the solar orbit that takes the spacecraft from Earth to Mars is called its trajectory.

Mars Via Least Energy Orbit

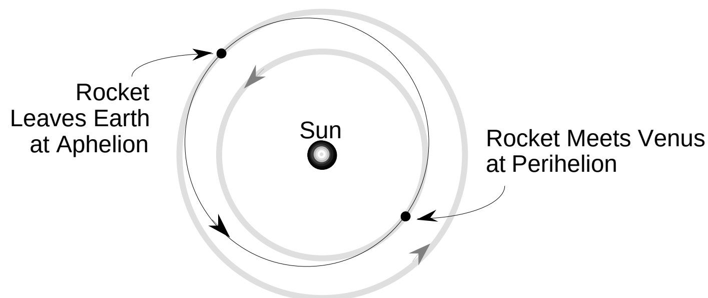


To achieve such a trajectory, the spacecraft lifts off the launch pad, rises above Earth's atmosphere, and is accelerated in the direction of Earth's revolution around the sun to the extent that it becomes free of Earth's gravitation, and that its new orbit will have an aphelion equal to Mars' orbit. After a brief acceleration away from Earth, the spacecraft has achieved its new orbit, and it simply coasts the rest of the way. To get to the planet Mars, rather than just to its orbit, requires that the spacecraft be inserted into the interplanetary trajectory at the correct time to arrive at the Martian orbit when Mars will be at the point where the spacecraft will intercept the orbit of Mars. This task might be compared to throwing a dart at a moving target. You have to lead the aim point by just the right amount to hit the target. The opportunity to launch a spacecraft on a transfer orbit to Mars occurs about every 25 months.

To be captured into a Martian orbit, the spacecraft must then decelerate relative to Mars (using a retrograde rocket burn or some other means). To land on Mars, the spacecraft must decelerate even further (using a retrograde burn, or spring release from a mother ship) to the extent that the lowest point of its Martian orbit will intercept the surface of Mars. Since Mars has an atmosphere, final deceleration may be performed by aerodynamic braking, and/or a parachute, and/or further retrograde burns.

To launch a spacecraft to an inner planet such as Venus using the least propellant possible, its existing solar orbit must be adjusted so that it will take it to Venus. In other words, the spacecraft's aphelion will be on Earth's orbit, and the perihelion will be on the orbit of Venus. As with the case of Mars, the portion of this orbit that takes the spacecraft from Earth to Venus is called a trajectory. To achieve an Earth to Venus trajectory, the spacecraft lifts off the launch pad, rises above Earth's atmosphere, and is accelerated opposite the direction of Earth's revolution around the sun (decelerated) to the extent that its new orbit will have a perihelion equal to Venus's orbit. Of course the spacecraft will end up going in the same direction as Earth orbits, just a little slower. To get to Venus, rather than just to its orbit, again requires that the spacecraft be inserted into the interplanetary trajectory at the correct time to arrive at the Venusian orbit when Venus will be at the point where the spacecraft will intercept the orbit of Venus. Venus launch opportunities occur about every 19 months.

Venus Via Least Energy Orbit

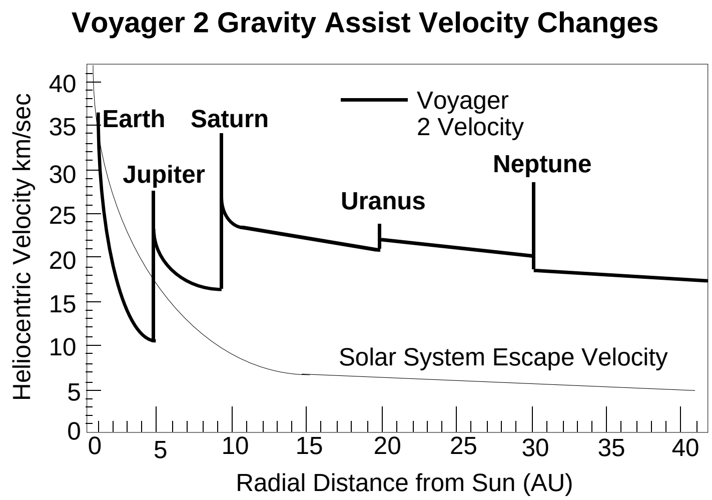


Gravity Assist Trajectories

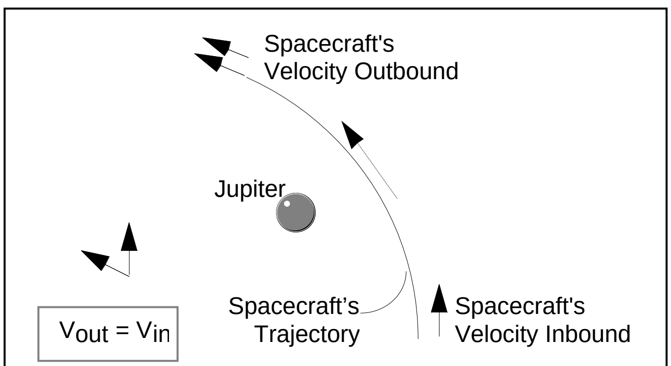
The first chapter pointed out that the planets retain the vast majority of the solar system's angular momentum. It is this momentum that is used to accelerate spacecraft on so-called "gravity-assist" trajectories. It is commonly stated in newspapers that spacecraft such as Voyager and Galileo use a planet's gravity during a flyby to slingshot it farther into space. How does this work? In a gravity-assist trajectory, angular momentum is transferred from the orbiting planet to a spacecraft approaching from behind. Gravity assists would be more accurately described as angular-momentum assists.

Consider Voyager 2, which toured the Jovian planets. The spacecraft was launched on a standard Hohmann transfer orbit to Jupiter. Had Jupiter not been there at the time of the spacecraft's arrival, the spacecraft would have fallen back toward the sun, and would have remained in elliptical orbit as long as no other forces acted upon it. Perihelion would have been at 1 AU, and aphelion at Jupiter's distance of about 5 AU.

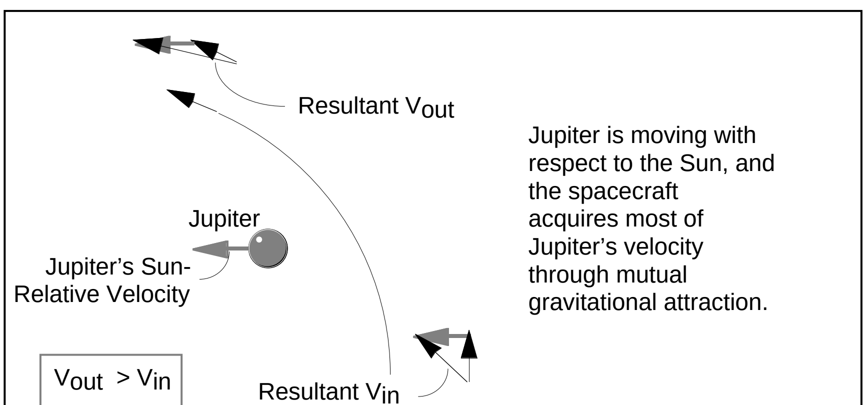
However, the spacecraft's arrival was carefully timed so that it would pass behind Jupiter in its orbit around the sun. As the spacecraft came into Jupiter's gravitational influence, it fell toward Jupiter, increasing its speed toward maximum at closest approach to Jupiter. Since all masses in the universe attract each other, Jupiter sped up the spacecraft substantially, and the spacecraft slowed down Jupiter in its orbit by a tiny amount, since the spacecraft approached from behind. As the spacecraft passed by Jupiter (its speed was greater than Jupiter's escape velocity), of course it slowed down again relative to Jupiter, climbing out of Jupiter's gravitational field. Its Jupiter-relative velocity outbound was the same as its velocity inbound. But relative to the sun, it never slowed all the way to its initial approach speed. It left the Jovian environs carrying an increase in angular momentum stolen from Jupiter. Jupiter's gravity served to connect the spacecraft with the planet's huge reserve of angular momentum. This technique was repeated at Saturn and Uranus.



The same can be said of a baseball's acceleration when hit by a bat: angular momentum is transferred from the bat to the slower-moving ball. The bat is slowed down in its "orbit" about the batter, accelerating the ball greatly. The bat connects to the ball not with the force of gravity from behind as was the case with a spacecraft, but with direct mechanical force (electrical force, on the molecular scale, if you prefer) at the front of the bat in its travel about the batter, translating angular momentum from the bat into a high velocity for the ball.



Relative to Jupiter, the spacecraft ends up with the same speed leaving as it had coming in, even though it accelerates during flyby.



The result is that the magnitude of V_{out} exceeds that of V_{in} relative to the Sun.

Gravity assists can be also used to decelerate a spacecraft, by flying in front of a body in its orbit, donating some of the spacecraft's angular momentum to the body. When the Galileo spacecraft arrived at Jupiter, passing close in front of Io in its orbit, Galileo experienced deceleration, helping it achieve Jupiter orbit insertion.

The gravity assist technique was pioneered by Michael Minovitch in the early 1960s. He was a UCLA graduate student who worked summers at JPL.

Recap

1. To launch a spacecraft to an outer planet such as Mars... its existing _____________ _____________ must be adjusted to cause it to take it to Mars.
2. The portion of the solar orbit that takes the spacecraft from Earth to Mars is called a __________________ .
3. To get to Mars, rather than just its orbit, will require that the spacecraft be inserted into the interplanetary trajectory at the correct _______________ .
4. To orbit Mars, the spacecraft must ________________ sufficiently... to be captured in a martian orbit.
5. In a gravity-assist trajectory, _________________ ___________________ is transferred from the orbiting planet to a spacecraft approaching from behind.

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1. solar orbit   2. trajectory   3. time   4. decelerate   5. angular momentum

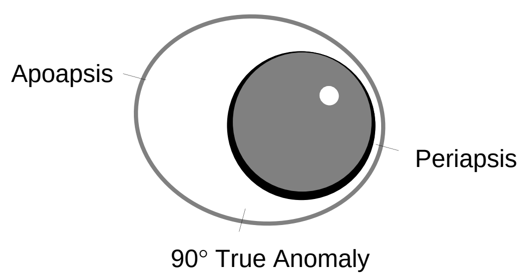
Chapter 5. Planetary Orbits

Objectives: Upon completion of this chapter you will be able to describe in general terms the characteristics of various types of planetary orbits. You will be able to describe the general concepts and advantages of geosynchronous orbits, polar orbits, walking orbits, sun-synchronous orbits, and some requirements for achieving them.

Orbital Parameters and Elements

The terms orbit period, periapsis, apoapsis, etc., were introduced at the end of Chapter 3.

The direction a body travels in orbit can be direct, or prograde, in which the spacecraft moves in the same direction as the planet rotates, or retrograde, going in a direction opposite the planet's rotation. True anomaly is a term used to describe the locations of various points in an orbit. It is the angular distance of a point in an orbit past the point of periapsis, measured in degrees. For example, a spacecraft might cross a planet's equator at 10° true anomaly. Nodes are points where an orbit crosses a plane. As an orbiting body crosses the ecliptic plane going north, the node is referred to as the ascending node; going south, it is the descending node.



To completely describe an orbit mathematically, six quantities must be calculated. These quantities are called orbital elements, or Keplerian elements. They are: Semi-major axis (1) and eccentricity (2), which are the basic measurements of the size and shape of the orbit's ellipse (described in Chapter 3. Recall an eccentricity of zero indicates a circular orbit). The orbit's inclination (3) is the angular distance of the orbital plane from the plane of the planet's equator (or from the ecliptic plane, if you're talking about heliocentric orbits), stated in degrees: an inclination of 0° means the spacecraft orbits the planet at its equator, and in the same direction as the planet rotates. An inclination of 90° indicates a polar orbit, in which the spacecraft passes over the north and south poles of the planet. An inclination of 180° indicates an equatorial orbit in which the spacecraft moves in a direction opposite the planet's rotation (retrograde). The argument of periapsis (4) is the argument (angular distance) of periapsis from the ascending node. Time of periapsis passage (5) and the celestial longitude of the ascending node (6) are the remaining elements. Generally, three astronomical or radiometric observations of an object in an orbit are enough to pin down each of the above six Keplerian elements.

Elements of Magellan's Initial Orbit at Venus

(1)	Semimajor Axis:	10434.162 km
(2)	Eccentricity:	0.2918967
(3)	Inclination:	85.69613°
(4)	Argument of Periapsis:	170.10651°
(5)	1990 Day of Year	222 19:54 UTC ERT
(6)	Longitude of Ascending Node:	-61.41017°
	(Orbital Period:	3.26375 hr)

Types of Orbits

Geosynchronous Orbits

A geosynchronous orbit (GEO) is a direct, circular, low inclination orbit about Earth having a period of 23 hours, 56 minutes, 4 seconds. A spacecraft in geosynchronous orbit maintains a position above Earth constant in longitude. Normally, the orbit is chosen and station keeping procedures are implemented to constrain the spacecraft's apparent position so that it hangs motionless above a point on Earth. In this case, the orbit may be called geostationary. For this reason this orbit is ideal for certain kinds of communication satellites, or meteorological satellites. To attain geosynchronous orbit, a spacecraft is first launched into an elliptical orbit with an apoapsis altitude in the neighborhood of 37,000 km. This is called a Geosynchronous Transfer Orbit (GTO). It is then circularized by turning parallel to the equator and firing its rocket engines at apoapsis.

Polar Orbits

Polar orbits are 90° inclination orbits, useful for spacecraft that carry out mapping or surveillance operations. Since the orbital plane is, nominally, fixed in inertial space, the planet rotates below a polar orbit, allowing the spacecraft low-altitude access to virtually every point on the surface. The Magellan spacecraft used a nearly-polar orbit at Venus. Each periapsis pass, a swath of mapping data was taken, and the planet rotated so that swaths from consecutive orbits were adjacent to each other. When the planet rotated once, all 360° longitude had been exposed to Magellan's surveillance.

To achieve a polar orbit at Earth requires more energy, thus more propellant, than does a direct orbit of low inclination. To achieve the latter, launch is normally accomplished near the equator, where the rotational speed of the surface contributes a significant part of the final speed required for orbit. A polar orbit will not be able to take advantage of the "free ride" provided by Earth's rotation, and thus the launch vehicle must provide all of the energy for attaining orbital speed.

Walking Orbits

Planets are not perfectly spherical, and they do not have evenly distributed mass. Also, they do not exist in a gravity “vacuum”—other bodies such as the sun, or satellites, contribute their gravitational influences to a spacecraft in orbit about a planet. It is possible to choose the parameters of a spacecraft’s orbit to take advantage of some or all of these gravitational influences to induce precession, which causes a useful motion of the orbital plane. The result is called a walking orbit or a precessing orbit, since the orbital plane moves slowly with respect to fixed inertial space.

Sun-synchronous Orbits

A walking orbit whose parameters are chosen such that the orbital plane precesses with nearly the same period as the planet’s solar orbit period is called a sun-synchronous orbit. In such an orbit, the spacecraft crosses periapsis at about the same local time every orbit. This can be useful if instruments on board depend on a certain angle of solar illumination on the surface. Mars Global Surveyor’s intended orbit at Mars is a 2-pm Mars local time sun-synchronous orbit. It may not be possible to rely on use of the gravity field alone to exactly maintain a desired synchronous timing, and occasional propulsive maneuvers may be necessary to adjust the orbit.

Recap

1. An inclination of zero means the spacecraft orbits the planet at its ___________, and an inclination of 90° indicates a ____________ orbit.
2. A spacecraft in ______________ orbit appears to hang motionless above one position on Earth.
3. _______________ orbits are high-inclination orbits, useful for spacecraft that carry out planetary mapping or surveillance operations.
4. To completely describe an orbit mathematically, _________________ quantities must be calculated.
5. A walking orbit whose... orbital plane precesses with nearly the same period as the planet’s solar day is called a __________ _____________________ orbit.

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1. equator... polar *2. geostationary* *3. polar* *4. six* *5. sun-synchronous*
