What Is a Control Chart?

A statistical tool used to distinguish between process variation resulting from common causes and variation resulting from special causes.
Why Use Control Charts?

- Monitor process variation over time
- Differentiate between special cause and common cause variation
- Assess effectiveness of changes
- Communicate process performance
What Are the Control Chart Types?

Chart types studied in this module:

- X-Bar and R Chart
- Individual X and Moving Range Chart
  - For Variables Data
  - For Attribute Data

Other Control Chart types:

- X-Bar and S Chart
- Median X and R Chart
- c Chart
- u Chart
- p Chart
- np Chart
Control Chart Decision Tree

Are you charting attribute data?  

- **YES**  
  - Use XmR chart for attribute data

- **NO**  
  - Data are variables data

  Is sample size equal to 1?  
  
  - **YES**  
    - Use XmR chart for variables data
  
  - **NO**  
    - For sample size between 2 and 15, use X-Bar and R Chart
# Elements of a Control Chart

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<th>Average</th>
<th>Range</th>
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Legend:

1. Title: ____________________________________
2. _________________
3. 2
Elements of a Control Chart
Constructing an X-Bar & R Chart

Step 2 - Collect and enter data by subgroup

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Enter data by subgroup in time sequence
Constructing an X-Bar & R Chart

Step 3 - Calculate and enter subgroup averages

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Enter the average for each subgroup
Constructing an X-Bar & R Chart

Step 4 - Calculate and enter subgroup ranges

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Average | 15.36 | 15.04 | 15.82 | 15.36 | 15.98 | 15.34 | 15.52 | 15.58 | 14.56 |

Range | 1.5 | 1.2 | 3.6 | 1.2 | 1.9 | 1.6 | 1.4 | 2.4 | 1.6 |

Enter the range for each subgroup
Constructing an X-Bar & R Chart

Step 5  - Calculate grand mean
Step 6  - Calculate average of subgroup ranges
Step 7  - Calculate UCL and LCL for subgroup averages
Step 8  - Calculate UCL for ranges
Step 9  - Select scales and plot
Step 10 - Document the chart
Constructing an X-Bar & R Chart

Step 9 - Select scales and plot
Constructing an XmR Chart

**Step 2 - Collect and enter individual measurements**

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Enter individual measurements in time sequence
### Constructing an XmR Chart

**Step 3 - Calculate and enter moving ranges**

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</table>

**Enter the moving ranges**
Constructing an XmR Chart

Step 4 - Calculate average of data points
Step 5 - Calculate average of moving ranges
Step 6 - Calculate UCL and LCL for individual X
Step 7 - Calculate UCL for ranges
Step 8 - Select scales and plot
Step 9 - Document the chart
Constructing an Xmr Chart

Step 8 - Select scales and plot
Constructing an XmR Chart

Step 10  - Check for *inflated* control limits

Step 11  - If inflated, calculate 3.144 times median mR

Step 12a - Do not recompute if 3.144 times median mR is greater than 2.66 times average of moving ranges

Step 12b - Otherwise, recompute all control limits and centerlines
Constructing an XmR Chart

Step 10 - Check for inflated control limits

Any point is above upper control limit of moving range

2/3 or more of data points are below average of moving range
(13 of 19 data points = 68%)
Step 12b - Constructing an XmR Chart

**Upper Plot**

\[
\begin{align*}
\text{UCL}_X &= \bar{X} + (3.144) \text{ (Median Moving Range)} \\
\text{LCL}_X &= \bar{X} - (3.144) \text{ (Median Moving Range)} \\
\text{Centerline}_X &= \bar{X}
\end{align*}
\]

**Lower Plot**

\[
\begin{align*}
\text{UCL}_{mR} &= (3.865) \text{ (Median Moving Range)} \\
\text{LCL}_{mR} &= \text{None} \\
\text{Centerline}_{mR} &= \text{Median Moving Range}
\end{align*}
\]
Control Chart Zones

UCL

ZONE A

ZONE B

Centerline

ZONE C

ZONE C

ZONE B

ZONE A

LCL

1/3 distance from Centerline to Control Limits
Rule 1 - Interpreting X-Bar & R Charts

UCL

Centerline

LCL

ZONE A

ZONE B

ZONE C

Out of Limits
Rule 2 - Interpreting X-Bar & R Charts

2 out of 3 successive values in Zone A
Rule 3 - Interpreting X-Bar & R Charts

4 out of 5 successive values in Zones A & B
Rule 4 - Interpreting X-Bar & R Charts

8 successive values on same side of Centerline
### Exercise 1
**Values of X-Bar and Ranges**

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EXERCISE 1
X-Bar & R Control Chart

Note: Solid lines represent the grid used in this module; dashed lines separate zones.
EXERCISE 2

Values of Moving Ranges

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EXERCISE 2
XmR Control Chart

Note: Solid lines represent the grid used in this module; dashed lines separate the zones in the upper plot.
EXERCISE 2
XmR Control Chart Revised for Inflated Limits

Note: Solid lines represent the grid used in this module; light dashed lines divide the zones in the upper plot.