Epistemology and Rosen’s Modeling Relation

William Dress
Oak Ridge National Laboratory, P. O. Box 2008
Oak Ridge TN 37831-6011
wbd@ornl.gov

Keywords: Complex systems, bizarre systems, modeling relation, pseudo science

ABSTRACT

Rosen’s modeling relation is embedded in Popper’s three worlds to provide an heuristic tool for model building and a guide for thinking about complex systems. The utility of this construct is demonstrated by suggesting a solution to the problem of pseudo science and a resolution of the famous Bohr-Einstein debates. A theory of bizarre systems is presented by an analogy with entangled particles of quantum mechanics. This theory underscores the poverty of present-day computational systems (e.g., computers) for creating complex and bizarre entities by distinguishing between mechanism and organism.

INTRODUCTION

Rosen’s modeling relation (MR) provides a powerful method of understanding and exploring the nature of the scientific method. Of course, the scientific method is itself epistemology in action. A decade ago Robert Rosen published an essay [1] on epistemology in honor of David Bohm. The explanation and use of the MR in that essay appeared as a revelation that evidently had great explanatory power and seemed to invite further development as an epistemological tool. The scientific method currently receives lip service but little practical understanding in the day-to-day lives of scientists. The method seems to exist more as a topic of intellectual discourse than a guide to pragmatic behavior even though it has been discussed at length by many philosophers, including Karl Popper [2]. A clear and simple model of the scientific method contributes a clarity that volumes of philosophy cannot provide.

To provide an appropriate conceptual setting for the extended MR developed here, the necessary background and vocabulary are first presented. Popper’s 3 worlds are then suggested as a “container” for the MR. The ensuing framework allows one to analyze the MR itself, seeing how its parts fit onto the world of organisms, objects, brains, and theories—that is, the exceedingly complex, natural world comprised of all those atoms in their remarkable manifestations “of ships and shoes and sealing wax and cabbages and kings.”

As examples of embedding the MR in the 3 worlds, Popper’s problem of demarcation between science and pseudo science is revisited. The famous Einstein-Podolsky-Rosen paper serves to introduce the use of the MR as a tool for thinking. Finally, a mathematical definition of “bizarre” systems, based on a real-world bizarre object, is suggested. The definition meets our intuitive requirements of what such concepts as “bizarre” and “complex” should mean.

EPISTEMOLOGY OF THE MODELING RELATION

While the 3-worlds’ embedding of the MR presented in this section is hardly a necessary undertaking, it certainly provides a useful and convenient means of concept bookkeeping, allowing one to maintain clear thought processes and keep ideas disentangled.

Rosen's modeling relation

For those not familiar with Rosen's modeling relation [3], the following words are borrowed in a modified form from a previous publication [4] and summarize. For a canonical and detailed explanation of the MR, one can do no better than read Rosen's original works.

Without worrying about the niceties of meaning and existence, terms which a physicist or engineer takes for granted anyway, consider a natural system denoted by ***N*** and a corresponding formal system denoted by ***F***. The natural system consists of components where any identification, language, or discourse leads us immediately into some formal system. ***N*** not only exists, but exhibits behaviors that can be observed by the process of measurement [5]. The temporal evolution taking place in such an ***N*** is an internal entailment termed 'causality.' ***F*** likewise has components in the form of elements, perhaps axioms or theorems, or maybe just embryonic concepts bearing quasi-logical relationships to one another. The entailment structure possessed by ***F*** often takes the form of production rules that may be identified with inference of a logical, mathematical, or algorithmic nature. This entailment process is an internal operation within ***F*** that mirrors the autonomous, causal, and dynamic processes in ***N***.

The measurement process is subsumed by the concept of encoding behaviors of ***N*** into elements or sets in ***F***. Inferences are then be made in ***F*** by 'turning' the mathematical 'crank.' The results of the inferences must then be compared to the future behavior of ***N***, which process is called 'prediction' in science. Decoding is a dual process to encoding—one that the scientist uses to verify predictions by another measurement procedure on the future behavior of ***N***. The intimate relationship between the MR and the scientific method alluded to above should now be clear.

The possibility of making predictions and their subsequent verification lead to the idea of connectives or a relation between ***N*** and ***F*** that allow comparison of the two systems. The result, when all the pieces are placed in relationship to each other, is Rosen's modeling relation, shown in Figure 1.

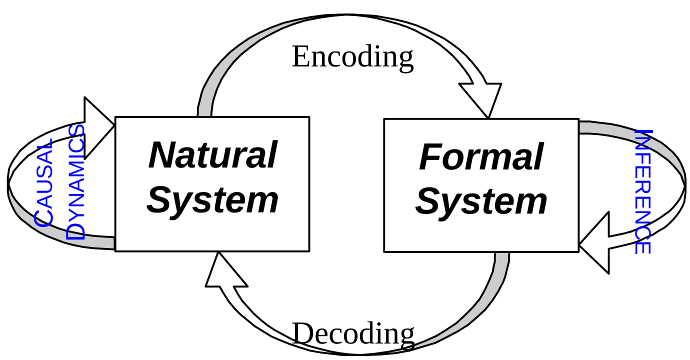

Figure 1. Block diagram of an abstract modeling relation wherein a natural system is modeled by a formal system. Each system has its own internal entailment structures and the two systems are connected by the encoding and decoding processes.

Of course, natural systems can model other natural systems and formal systems other formal systems. That is the power of the MR as a conceptual tool and the basis for talking about a mathematics of thought and, perhaps, consciousness.

Connection to Popper's 3 worlds

Popper himself gave the clearest expression of his 3-worlds' concept in the following passage:

> "[W]ithout taking the words 'world' or 'universe' too seriously, we may distinguish the following three worlds or universes: first, the world of physical objects or physical states; secondly, the world of states or consciousness, or of mental states, or perhaps of behavioral dispositions to act; and thirdly, the world *of objective contents of thought*, especially of scientific and poetic thoughts and works of art."—Karl Popper, [6, p. 59]

This concept of three worlds provides a convenient framework for the modeling-relation triad of natural system, encoding and decoding, and formal system. ***N*** and its components clearly belong to world 1 and the encodings and decodings as processes belong to world 2 while being conceptualized, and thence to world 3, as formal procedures. ***F*** is clearly a world 3 construct, while the inferences are world-2 activities or world-3 implications depending on who or what is performing them. Implications in the world-1 system—that is, causality—remain strictly a world-1 activity or relation.

Just as one would claim objective existence in world 1 for any particular ***N***, Popper would say that ***F*** likewise has objective existence—in world 3. Holding this distinction in mind can materially aid one's thought process when wrestling with new or complex ideas; confusing the worlds 1 and 3 leads to all sorts of nonsense and philosophical errors, primarily the so-called category error. Note that Popper does not suggest that any particular formal system must be correct or complete to be a candidate for world-3 existence—it inhabits world 3 merely by its being conceived and discussed. It takes a mind to bring a any world-1 object into world-3 existence; this observation contains a lovely circularity in that such an act of creation can only be accomplished by a world-1 organism. Popper argues convincingly [7] for the existence of world 3 and its creation by the human mind.

The modeling relation as epistemology

Given Rosen's concept of the modeling relation and its embedding in Popper's three worlds, the role of the MR in constructing a rational epistemology becomes clear. The canonical example of this marriage is the scientific method. Here, all three elements of the MR (the two systems, natural and formal, the entailment processes within each, and the encoding and decoding connectives) are identified with each of the three worlds. The only piece that is not a direct mapping is the act of encoding and decoding; while obviously world-2 activities, these connectives of the MR have referents in both world 1 and 3 and consequences in world 3. This "foot in both camps" nature of the connectives will be used below to disentangle the conceptual mess engendered by the Bohr-Einstein debates.

MR3 as a "framework" for thinking

Semiotics, as developed primarily by Peirce [8], is commonly utilized as a framework for discussing complex systems and even the process of thinking. However, as argued elsewhere [4], unless one's interest is the linguistics of signs, symbols, and meaning, the MR, particularly as developed by Rosen, is conceptually simpler to apply, more powerful in its interpretation, and broader in its range. A science of thinking and consciousness is better suited to our needs than is a science of linguistics, no matter how interesting or potentially useful.

The comprehensive nature of MR3, a term indicating the marriage of Rosen's MR and Popper's 3 worlds, accommodates both linguistics and semiotics yet extends far beyond either in providing a framework for discussing organisms, which was Rosen's original purpose. MR3 also accommodates any conceptual or natural system that can be thought of as a proper subject matter for science. In some sense, this is an enlargement of the scientific method from its narrow domain of natural science to the entire field of human intellectual activity.

BRAINS, MACHINES, AND A MATHEMATICS OF CONSCIOUSNESS

It is well known that the first complex organism (beyond yeast) to have its genome completely mapped is *C. Elegans*, the nematode. Well before this genome work was completed, others had mapped out the nematode’s central nervous system. In particular, a recent special report in Science [9] notes that there are a mere 302 cells in the worm’s nervous system and that many, if not most, of the neurotransmitters, receptors, and transmitter channels are already known. The complete “wiring diagram” of the neural connections has been worked out, as a mechanism, and it is not too difficult to imagine that electrical and chemical signals can be assigned to particular paths and activities. In short, it is almost certain that the entire “brain” of *C. Elegans* can and will be simulated in great detail on a super computer. The simulation can then “run” and be made to exhibit behavior arbitrarily close to that an actual creature. Since the model of the nematode’s brain—a model extracted from the physiology and biochemistry—will then be a simulation, the result can be nothing more than a simulation of a mechanism no matter how complicated and detailed it is made. Can the computer then be said to contain or generate or be an artificially live nematode? Only by greatly stretching any possible metaphorical interpretation the researchers will choose to give to the bit patterns that emerge, would such an assertion make sense. The point is that the simulation imagined here is just a “model” of an abstracted formal model and, as Rosen has shown [10], the entailment relations between the original model and the natural system are completely lost when passing to a simulation.

On analyzing the modeling relation

Above, it was suggested that natural systems can serve as models of both formal and natural systems. Likewise, formal systems can model other formal systems. If one analyzes (deconstructs) the modeling relation, it is possible to speak about models themselves, recognizing the danger in losing track of the referent. Suppose, for a moment, that the formal system ***F*** is disconnected from the natural system ***N***, leaving the encoding and decoding relations dangling. One might think of a molecule with a dangling orbital in search of another molecule with which to bond. To continue this metaphor, suppose there is a soup or solution containing many different types of ***F***s, each with a different “affinity” for the target system. As the solution is cooled, a “bond” will form with some other ***F***. Repeating this operation many times, many different bonds will form. The set of formal systems {***F***} that bond to the original ***F*** form an equivalence class with “temperature” as a granularity parameter, perhaps.

*Partial ordering on the set {**F**}*

Not pushing this metaphor too far, suppose that {***F***} in a given equivalence class can now be analyzed by an ordering relation. Conceptually construct a sequence of formal models, each one more “distant” from the original natural system by some measure of similarity or correspondence.

Since one theory or explanation can often said to be better than another by some established set of criteria, ordering of some sort does seem useful while ‘distance’ may be too strong a term at this stage—it may be that a metric space is too restrictive. Ordering does make sense as the notion of equivalence class is involved and one can certainly imagine equivalence classes of models. A paradigm for these ideas is to be found in the practice of science—returning again to the scientific method, some theories or models are better than others, the criteria being Occam’s razor, explanatory power, and correspondence of predicted behaviors with the natural system.

An essential step in a formal development of these ideas is to construct a space of formal systems. A partitioning into equivalence classes generated by a suitable relation on this space create conceptual objects each of which is either “bonded” to a particular ***N*** or not. Perhaps adding a “null” natural system, so that all equivalence classes are bonded, makes sense. The study of such augmented modeling relations then requires the full formalism of category theory.

The “floating” formal system

Suppose no member of {***F***} ever “bonds” strongly to any ***N***—the {***F***} can said to be “floating.” What are the properties of such floating formal systems? In one view, they seem to refer to pseudo science in the sense that any ***F*** in {***F***} not only does not refer to or closely correspond with any known ***N***, but the pseudo-***N*** it is presumably bound to also “floats” in that no expenditure of effort can clearly identify it. The sought-for ***N*** becomes a moving target; the set {***F***} is a set in search of a referent. Such a floating set cannot be dismissed outright as it may be an essential step in the creative process and an aid to speculations and brain-storming. This situation is not necessarily an error; just because a particular instance of the set is floating does not mean that future representations or some evolution of the set can’t find a suitable ***N***.

RESOLVING CONFLICTS AND PUZZLES

In this section, the above ideas are made more concrete by discussing some particular examples. These illustrations address primarily the use of MR3 as a framework for thinking. However, the examples should also hint at the direction that a formal theory of MR3 might take.

Demarcation and the problem of pseudo science

The problem Popper originally set for himself was to establish a demarcation between science and pseudo science (PS). Any good philosopher categorically rejects appeals to authority such as, “that is science, this is not (period).” Popper sought a natural and consistent way to classify methods and belief systems; the result was the criterion of falsification. Given today’s wide-spread cultural interest in such things as angels, astrology, aliens, creationism, *The X Files*, *Men in Black*, and so on and on and on, it seems that we have much the same problem nearly three-quarters of a century later. To compound matters, many pseudo sciences, or activities that we feel should be classified as such, make use of a modeling relation in much the same way as does actual science. Thus, PS can even be said to follow the scientific method in some perverse way. However, the ‘natural systems’ of PS do not necessarily exist in world 1—that is, they are imaginary constructs mostly in world 3. As such, the formal systems of PS are usually, but not entirely, incomplete systems in the sense of having only certain paths (implication chains) explored or publicized, or comprising inconsistent axioms or assumptions as well as having axioms or assumptions that are clearly at odds with the larger body of formal systems (e.g., science) that have been developed, amended, tested, and explored over the decades.

A method of demarcation suggested here is to identify precisely what the PS is talking about—not only what the object of the decoding process is claimed to be, but precisely how and from which natural system, if any, the encodings are said to have arisen. If there is a chain of formal systems, as is usually the case in intellectual constructs, simply chase along the chain in an attempt to locate some natural system that is the ultimate referent. In this view, France’s N-rays, Russia’s poly water, and Utah’s cold fusion are not examples of pseudo science since the chain of quasi-formal systems in all these cases could be followed and no ultimate natural system was found at the end. These models were simply in error. The claims of homeopathy, creationism, astrology, and many other systems have likewise been chased down the formal-system chain. The end result usually uncovers an appeal to authority, a circular sequence of formal systems, or simply an infinite regression of formal systems. Each of these endpoints indicates a PS.

Quantum mechanics and the EPR paradox

Recent activity in the rapidly expanding field of quantum optics and quantum measurement has led to questions concerning the locality of phenomena and the concept of causality as interpreted in quantum mechanics. Asking whether certain behaviors are possible and whether they can be used for practical applications in communications and sensing, one is led to revisit the famous Bohr-Einstein debates concerning the nature of quantum mechanics and what quantum mechanics can and cannot say ("Speakables and unspeakables" in Bell's terminology [11]). In 1935, Einstein, Podolsky, and Rosen (EPR) published a paper [12] that was to affect the way future generations of physicists thought about reality.

Einstein's viewpoint

Consider the opening paragraph of the famous EPR paper:

> "Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves."—Einstein, Podolsky, Rosen [12]

This is an expression of clarity that few have achieved so surely and succinctly; it evokes an image much like that of Fig 1. The statement clearly expresses the relationship between epistemology and ontology and, incidentally, suggests that Einstein's concern was with the ontological. Theory, for him, was a means to know reality, with the emphasis on 'reality.'

The paper then goes on to suggest a thought experiment where two particles are made to share their properties jointly. A measurement on one of the particles simultaneously provides knowledge of the shared state of the other. It wasn't until some thirty years later that such experiments became feasible, realizing Einstein's deepest concerns.

Bohr's viewpoint

Consider the following excerpt from Bohr's reply [13] to the EPR paper:

> "Of course there is in a case like that just considered [referring to the measurement on one member of a correlated 'EPR' pair allowing an inference to be made concerning the other member] no question of a mechanical disturbance of t he system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.*"—Nils Bohr

Bell [11] thought this incomprehensible and it certainly seems so, even after much pondering. However, when viewed in context of Einstein's statement and taking a hint from Jaynes [14], who "translated" this quote into "plain English," it is evident that Bohr's primary concern was one of epistemology. Bohr insisted that the referents of quantum mechanics were beyond our ken other than through the epistemological apparatus of the theory. The contextual interaction of quantum entities with the apparatus is all that one can know and is perfectly well described by the theory. Theory, for Bohr, was the reality since the ontological lay outside of what we can know (by direct apprehension?) and we should not (as a moral principle?) even ask questions concerning what might lie beyond the epistemological. In a sense, Bohr restricted his epistemology to the right-hand side of the MR in Fig. 1, at the expense of the entire diagram. Analyzed in this fashion, Bohr's reply to Einstein's challenge becomes fully comprehensible.

Resolution

Although the debate initiated by Einstein continues to this day, a resolution to this paradox can be constructed that closely follows Jaynes’ ideas and makes explicit use of Rosen’s modeling relation. Consider the following particular modeling relation between a natural system (quantum reality) and a formal system (quantum mechanics). The encodings (measurements) and decodings (predictions) are present as are the causal dynamics, which can only be inferred but never perceptually apprehended. The inference or entailment structure of proofs of theorems and calculations are also indicated. This situation is shown in Figure 2.

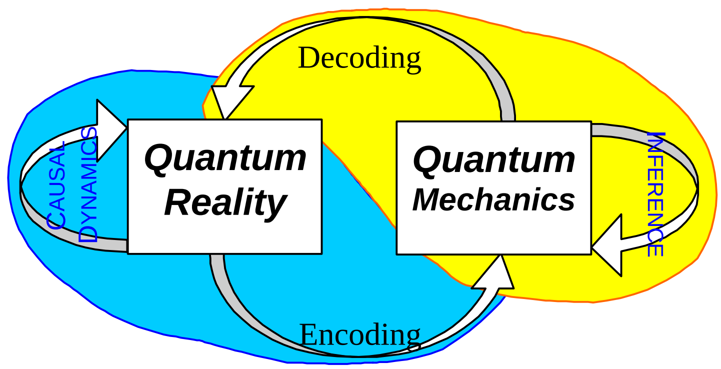

Figure 2. The Bohr-Einstein debates seen from the viewpoint of the modeling relation.

Not taking the dualistic metaphor too seriously, the upper shaded yang region represents Bohr’s point of view, while the lower (and hidden) yin region is Einstein’s. These regions of focus or interest, when overlaid on Rosen’s MR, clearly show that the whole debate arose when two people viewed two different aspects of the same whole—the modeling *relation*.

BIZARRE SYSTEMS

The idea of a “bizarre” system is a slippery thing to discuss in general terms. Fortunately, there are examples to be found in both the mathematics of quantum physics and in actual experimental situations that can only be described by the term “bizarre.” These objects are presented as simple examples of bizarre systems by developing the mathematics of Hilbert spaces enhanced with several physical postulates that turn the abstract formal system into a model of quantum reality. The formal system presented here is a subset of standard quantum mechanics, which is known to be both correct and highly accurate in its description of physical reality (the natural system) on a quantum level. The suggestion, then, is to extend the concept of a bizarre system from these known objects, using the same general mathematics to be able to talk about bizarre systems in the world of complex systems and organisms. It is not necessary to retain the concept of Hilbert space, but it is be necessary to use sets and, eventually, categories. In some sense, a hierarchy of models is suggested, starting with the simple quantum system of the biphoton, which is mathematically equivalent to a pair of “magic” coins or dice that are entangled in a particular Hilbert space.

Mathematical background

Define *Hilbert space* as a complete, linear, inner-product vector or function space over the complex numbers. Using Dirac notation, the inner product of two vectors $|a\rangle$ and $|b\rangle$ belonging to the Hilbert space, $\boldsymbol{H}$, is represented by the symbols $\langle a|b\rangle$. This is equivalent to the usual notation for the inner product: $(a, b) = \int a^*(x)b(x)dx$ where the integration is over the domain of definition of the functions $a(x)$ and $b(x)$, and * denotes complex conjugation. On discrete spaces, the integral is replaced by a sum. Suppose and $|y\rangle \in \boldsymbol{H}$ and $\{|f_n\rangle\}$ is an orthonormal basis for $\boldsymbol{H}$. Then an expansion of $|y\rangle$ in the f-representation is given by

$$|y\rangle = \sum_j c_j |f_j\rangle \tag{1}$$

for a particular set of complex numbers $\{c_j\}$. Take the inner product of $|y\rangle$ with $|f_m\rangle$ and observe that $c_m = \langle f_m|y\rangle$ since $\langle f_i|f_j\rangle = d_{i,j}$, which is the Kronecker symbol that is 1 for $i = j$ and zero otherwise. Assume that $|y\rangle$ is normalized, that is $\langle y|y\rangle = 1$. In the f-representation, that normalization may be written, noting that a complex number becomes its complex conjugate in the adjoint space, as

$$\langle y|y\rangle = \sum_{j,i} c_j^* c_i \langle f_j|f_i\rangle = \sum_{j,i} c_j^* c_i d_{j,i} = \sum_j |c_j|^2; \tag{2}$$

but this expression is equal to 1 since the function is normalized. In addition, note that the absolute value of any number is   0.

Define (*Bayesian*) *probability* as a mapping, *p*, from a denumerable set of statements X to the interval [0,1] such that $p(x)$ 0, $p(\emptyset) = 0$, and $p(X) = 1$ where $x \in X$. $p(x)$, read “the probability that statement x is true” or , simply, “the probability of x,” is always assumed to be *conditioned* on some set of circumstances such as an experimental arrangement. This conditioning may be explicitly given by the notation $x|y$ which is read “x given that the statement y is true.” The mapping p obeys the product and sum rules [16], which state that $\mathrm{p}(xy) = \mathrm{p}(x)\ \mathrm{p}(y|x)$ and $p(x+y) = p(x) + p(y) - p(xy)$, where $x,y \in X$, “$x+y$” is the disjunction of the two statements and “xy” or “x,y” is their conjunction. Note that both conjunction and disjunction are associative and commutative and that conjunction distributes over disjunction as in ordinary arithmetic. Since the spaces of interest are finite dimensional, as they must be for any real-world example of interest, the full machinery of (Kolmogorov) probability theory is not required.

A central postulate of quantum mechanics, which we require to talk about bizarre quantum systems, is that any physical observable of a quantum system is represented by a linear, self-adjoint operator on the corresponding Hilbert space. An operator A is self-adjoint if $\langle u|Av\rangle = \langle Au|v\rangle$ for any two non-zero vectors belonging to $\boldsymbol{H}$. The other postulate needed identifies a probability distribution with amplitudes of normalized vectors expanded in an eigenbasis of any self-adjoint operator on the Hilbert space in question; “amplitude” refers to the coefficient in such an eigenexpansion, which are the c_j in (1). The postulate states that the square of the absolute value of an amplitude of a normalized vector in a Hilbert space is the probability of obtaining the eigenvalue belonging to that eigenvector in a measurement of the corresponding operator.

Magic coins and dice

With these definitions, consider a two-sided die (coin) on a real vector space. This object can be represented by the column vectors $|heads\rangle = (1,0)^{\mathrm{T}}$ and $|tails\rangle = (0,1)^{\mathrm{T}}$ where T means transpose. The “which-side” operator is the diagonal matrix with the number of spots (say 1 and 2 where 1 means “heads” and 2 “tails”) along the diagonal. The state describing the coin during a toss is $(a, b)^{\mathrm{T}}$ where $a^2+b^2 = 1$ and both amplitudes a and b are real. A fair coin has $|a| = |b|$ since negative numbers are allowed; and the probability of heads is $|a|^2$ and tails $|b|^2$. This simple example allows one to compute the probabilities in any coin toss by identifying the square of the amplitudes with the probabilities of obtaining either heads or tails. So far, no magic or bizarre behavior.

The next step is to consider two coins or dice, each represented on its own Hilbert space. How can these entities be combined? For linear spaces, there are only two ways of describing the two coins that both make sense and are not unduly complicated: on a direct sum space and on a direct product space. Both combinations are defined using the notion of a Cartesian product. The *direct (Cartesian) product* of two vector spaces $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ over the same field (written as $\boldsymbol{V} = \boldsymbol{V_1} \otimes \boldsymbol{V_2}$) is the set of pairs$\{(x_i, y_j)$: $x_i \in \boldsymbol{V_1}$, $y_j \in \boldsymbol{V_2}\}$ such that $a(u, v) = (au, av)$ and $(x, y)+(u, v) = (x+u, y+v)$ where a is an element of the scalar field. In the Dirac notation chosen, simply choose one from each space, as $|i\rangle|j\rangle$ where $|i\rangle \in \boldsymbol{V_1}$ and $|j\rangle \in \boldsymbol{V_2}$. The order in which the two vectors are written distinguishes equally well between the two spaces as does the (., .) notation, not to be confused with the alternate notation for an inner product; if we need to specify the particular factor space, then subscripts can be used as $|i\rangle_1|j\rangle_2$. If the spaces are finite-dimensional Hilbert spaces, it is easy to prove that their product is also a Hilbert space. An eigenbasis for the product space is the product of two separate eigenbases. A typical vector in the product space is written as

$$|y\rangle = \sum_{i,j} c_{i,j} |i\rangle |j\rangle, \tag{3}$$

which adequately describes the state of two dice or coins tossed together; it also describes quantum objects such as two electrons or two photons or even an electron and a proton. Given such a state vector as (3), one can now ask questions about joint observations in a toss of coins, dice, or even jointly observing the spin of two electrons or the polarization of two photons. The expression for the joint probability, in a single toss of a pair of dice, of observing eigenvalue m for the first die and eigenvalue n for the second is given by

$$p(m,n) = |\langle m|\langle n|y\rangle|^2 = \left|\sum_{i,j=1}^{k} c_{i,j} \langle m|i\rangle_1 \langle n|j\rangle_2\right|^2 = |c_{m,n}|^2, \tag{4}$$

where k is 6 for dice or 2 for coins.

What about the other way of combining the two spaces? If the two spaces are disjoint, such as the space of all left-handed gloves and the space of all right-handed gloves, then the direct sum can be defined analogously with the above definition of a direct product. However, the direct-sum space is a union of the two disjoint spaces (it is the space of all pairs of gloves in this example, one glove from the left space, the other from the right space). A typical member of this space (describing a pair of gloves, for example) is

$$|g_{pair}\rangle = a|g_l\rangle + b|g_r\rangle, \tag{5}$$

which generates the same general joint probability table as (4) would give for a pair of coins (k=2). How is one to decide which formulation is correct? Are we dealing with gloves (direct sums) or coins (direct products)? As usual in such cases, one appeals to experiment. Dozens of careful experiments done over the past three decades have unequivocally shown that the answer is “coins.” The product-space formulation is consistent with experiment; the sum space is not.

So what? What is bizarre about this? Consider the combined function or state of two photons (or coins) on a product Hilbert space using (3) with $c_{1,1} = c_{2,2} = 0$ and $c_{2,1} = c_{1,2} = 1/ 2$. One can ea s-ily convince oneself that such a system is fair by computing a joint probability table and then summing rows to obtain the marginal probability for one coin and columns to obtain that of the other. From (4), the joint probability can be represented as a 2 by 2 matrix with zeros on the diagonal — (heads, heads) and (tails, tails)—and 1/2 for the other two entries. The probability that any given coin shows heads is equal to 1/2 as is the probability for tails. However, the combinations (heads, heads) and (tails, tails) never appear. This is truly bizarre! Suppose one tosses the two coins such that one lands first, showing heads. One can predict *with certainty* that the other *will* show tails *when* it lands first, showing heads. One can predict *with certainty* that the other *will* show tails *when* it lands, yet the coins are fair. Such coins are said to be "magic" [17].

Real-world 'magic' coins

The same situation obtains for electrons or photons or any other two quantum entities that are properly prepared in a source that emits them together as pairs. Such pairs are said to be "entangled," which simply means that their description as a function on the product Hilbert space cannot be factored into a product of two functions, one belonging to each individual space. If each $c_{i,j}$ in (3) were equal to 1/2, such a factorization would be possible and the joint probability matrix would be 2 by 2 with 1/4 for each of the 4 entries. Entanglement is not possible on a sum space.

The *point* of this mathematical digression into quantum physics is to make clear the distinction between sum and product spaces; the former do not support bizarre behavior while the latter do. This observation leads to the meaningful and reasonable suggestion that the term "bizarre," when used in conjunction with complex systems and artificial organisms, might profitably refer to an analysis on direct product spaces. The *motivation* for the distinction is that such bizarre systems *actually exist* in the quantum world. There is no reason why they cannot exist in the macro-world referred to in the Introduction. The suggestion here is that they indeed exist and are called *organisms*.

Speculation on the possibility of artificial organisms

That such a simple system as described above can exhibit truly bizarre behavior gives us hope that our quest for artificial organisms might not be in vain. I believe we can learn how to construct complex, intelligent, and perhaps even conscious artifacts by a careful and direct extrapolation of the binary behavior of entangled quasi coins such as electrons and photons.

Protein synthesis is a quantum-mechanical process taking place within cells, both eukaryote and prokaryote. Biological macromolecules involved in this process are in well-defined quantum states and, ignoring thermalization processes demanded by equilibrium thermodynamics [14], these states are shared during interactions between molecules. Upon separation, when the protein leaves the ribosome, the entanglement is maintained even though it will be gradually lost due to an environmental-interaction effect known as "decoherence." Suppose the protein can reach the cell wall before full decoherence; here, there may be several possibilities such as opening an ion channel, passing through the membrane, or other bio-functions involving the membrane or other cellular structures. Suppose, further, that one of the protein's entangled partners also has a range of possibilities of interaction in its own neighborhood, possibilities that have *non-uniform* probabilities. Then, by an argument based on the probabilistic behavior of functions on the product Hilbert space (of very *large* dimensions), a quantum-mechanical calculation forces one to conclude that, *in probability*, the interaction occurring in the cell's interior will *determine*, in the sense used above, the particular choice made by the protein as it interacts with the cell wall. The point is that "decisions" made at the cell center can influence actions at the periphery without a *mechanism* being present. A disrupted cell would be open to excessive decoherence, effectively putting an end to its normal functioning; the cell would experience death. *Only* if the disruption could be removed in such a way as to re-establish quantum entanglement could the cell live again.

Alternatively, *only* if we can construct artifacts that allow entanglement at the quantum level while maintaining the large-dimensional complexity required of living organisms, will we succeed in creating artificial organisms. It is only artificial *organisms* that will be able to exhibit—not simulate—life, intelligence, and consciousness; their entailment processes *must be* causal, not formal. The hope that present day computers can be "alive" or "conscious" or "spiritual" is based on mere naïve analogy with certain behaviors observed in real, live organisms and a false imputation of intentionality. Until we produce complex constructs based on quantum entanglement, such hopes will remain unrealized dreams.

EPILOGUE

The heuristic structure of MR3 (Rosen's modeling relation embedded in Popper's 3 worlds) has been proposed as a structure for clear thinking and a guide to building models of complex systems. Using such an aid, pseudo science becomes easier to identify. Additionally, the great quantum debate of the past 75 years is seen to be nothing more serious than the parable of the three blind men and the elephant—the resolution was to identify the elephant. Of course, there is much more involved than this glib abstraction, but each twist and turn in the conceptual windings of the arguments on both sides is amenable to an analysis via the modeling relation.

The bizarre behavior of quantum-mechanical "coins" or "dice" leads to an existential basis for understanding the "bizarre" behavior of complex system by analyzing such systems on product spaces instead of the sum spaces [10, 18] that commonly describe computers and algorithmic, formal processes. That there is such a thing as bizarre behavior gives us hope of constructing artificial organisms.

REFERENCES

1. Robert Rosen, "Some epistemological issues in physics and biology," in *Quantum Implications: Essays in Honor of David Bohm*, ed. F. David Peat and Basil Hiley, Routledge & Kegan Paul, 1987.
2. Karl R. Popper, *Conjectures and Refutations: The Growth of Scientific Knowledge*, Harper & Row, 1963. See also Popper's *The Logic of Scientific Discovery*, Harper & Row, 1959.
3. Robert Rosen, *Anticipatory Systems*, Pergamon Press, New York, 1985.
4. William Dress, "A Bayesian approach to extracting meaning from system behavior," *IEEE SMC '98 Conference Proceedings*, IEEE Press, 1998. Or on the web at http://vic.com/~wbd/Bayes.html
5. Robert Rosen, *Fundamentals of Measurement and Representation of Natural Systems*, Elsevier, 1978.
6. Karl R. Popper, *Popper Selections*, edited by David Miller, Princeton University Press, 1985.
7. Karl R. Popper and John C. Eccles, *The Self and its Brain,* Springer-Verlag, Chapter P2, 1977.
8. Thomas A. Goudge, *The Thought of C. S. Peirce*, Dover Publications, New York, 1950.
9. Special Section, *Science* **282**, pp. 2011-2046, 1998.
10. Robert Rosen, *Life Itself*, Columbia University Press, New York, 1991.
11. J. S. Bell, *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, New York, 1987.
12. A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?", *Phys. Rev.*, **47**, pp. 777-80, 1935.
13. Nils Bohr, "Can quantum-mechanical description of physical reality be considered complete?", *Phys. Rev.*, **48**, pp. 696-702, 1935.
14. E. T. Jaynes, "Clearing up mysteries—the original goal," *Maximum Entropy and Bayesian Methods*, edited by J. Skilling, Kluwer Academic Publishers, Dordrecht-Holland, pp. 1-27, 1989.
15. P. C. W. Davies and J. R. Brown, *The Ghost in the Atom,* Cambridge University Press, 1986.
16. R. T. Cox, Probability, Frequency and Reasonable Expectation, *Am. J. Phys.*, **14**, (1946) 1-13.
17. William Dress, "'Magic' Quasi Dice," *Amer. Math. Monthly*, to be published.

18. Donald C. Mikulecky, "Robert Rosen: the well posed question and its answer—Why are organisms different from machines?," to appear in *Proceedings of the 43rd Annual Meeting of the International Society for the System Sciences*, 1999.