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Adaptive Peircean Decision Aid Project Summary Assessments

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Abstract

This efforts objective was to identify and hybridize a suite of technologies enabling the development of predictive decision aids for use principally in combat environments but also in any complex information terrain. The technologies required included formal concept analysis for knowledge representation and information operations, Peircean reasoning to support hypothesis generation, Mill's canons to begin defining information operators that support the first two technologies and co-evolutionary game theory to provide the environment / domain to assess predictions from the reasoning engines. The intended application domain is the IED problem because of its inherent evolutionary nature. While a fully functioning integrated algorithm was not achieved the hybridization and demonstration of the technologies was accomplished and demonstration of utility provided for a number of ancillary queries.

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Abstract

This efforts objective was to identify and hybridize a suite of technologies enabling the development of predictive decision aids for use principally in combat environments but also in any complex information terrain. The technologies required included formal concept analysis for knowledge representation and information operations, Peircean reasoning to support hypothesis generation, Mill's canons to begin defining information operators that support the first two technologies and co-evolutionary game theory to provide the environment / domain to assess predictions from the reasoning engines. The intended application domain is the IED problem because of its inherent evolutionary nature. While a fully functioning integrated algorithm was not achieved the hybridization and demonstration of the technologies was accomplished and demonstration of utility provided for a number of ancillary queries.

1. Introduction

The effort described in this report captures observations and assessments of an R&D effort to put greater theoretical foundations to efforts in the information domains. Significant levels of effort have gone into designing information systems without the fundamental theoretical foundations needed to build robust information systems. As a result I see systems that lack the requisite logics needed to manipulate or operate on information, the logics needed to interpret, update, secure or process the massive amounts of information we need to deal with the problems of command on network centric battlefields, to deal with the terrorist threats against our homeland or even the information associated with remaining economically competitive.

This effort attempted, with too little time and money to address the issues and demonstrate a solution focused on the IED terrorist threat. The objectives were to identify and hybridize a suite of technologies to provide a system to provide unobtrusive predictive decision support for a commander in the field. Linking mathematically robust knowledge representation to a Peircean based reasoning engine and then integrating that into a co-evolutionary game environment we could produce a system with a capability of anticipating the next moves of an adversary without having to wait for a statistically significant pattern of destruction to emerged before we define a counter strategy. Approaches founded on deductive and inductive decision aids can only react to situations on the ground, which in a highly dynamic environment only lead to unacceptable losses.

The effort initiated suffered from many excursions to address questions of applicability of different subsets of technology on specific questions associated with information problems. What I will try to do in this report is identify some of the application areas and how the different groupings of technology can address these unique problems. One observation as a result of this effort involves the linkage between theory and engineering. What I see is the potential for decades of potential theoretical research to flesh out all the potential of these technologies while from an engineering perspective realizing that the 80% solution will advance capabilities beyond anything we have in the field or will have in the field in the next 20 years if we pursue traditional piecemeal approaches being pursued today.

2. Structure of Adaptive Decision Aids

The first step in identifying the needs of a command decision support sub-system is to understand the decision making process. It is felt that we often neglect the cognitive load imposed on our commanders and as a result provide them with burdensome applications that take away from a fundamental task, one of survival. Systems engineering provides the means by which we can assess the larger context of the problem being addressed to ensure we solve the correct problem. One observation in the process is the need to understand the decision making process from a philosophically based perspective, and to approach the design in a manner that augments the decision making process and mitigates the impact on the tasks being addressed. Recognizing that decisions are based on a decision makers "belief state" enables us to design decision aids that simply modify that belief state.

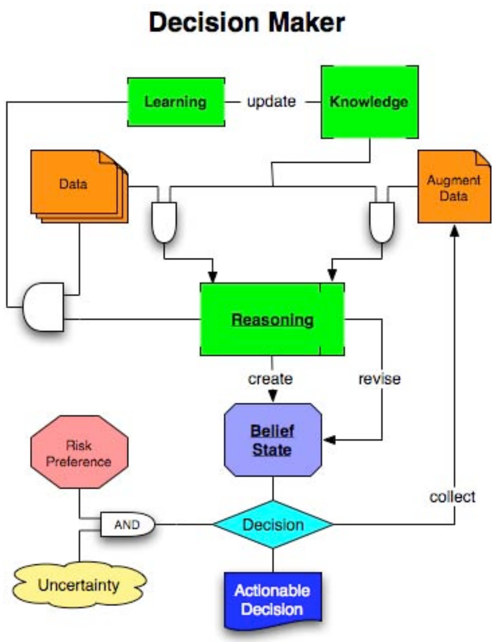

Figure 1. Decision making paradigm.

The model shows a system that collects data and convolves that with their collected knowledge to create an understanding of a situation, creating a belief state. The model permits the accretion of more data as well as updating the knowledge base, through learning or by adding to the command collective, individuals or systems with different skills. Once a belief state is generated decisions are made which are tempered by uncertainty, and risk aversion. This model also adds some insight into the concept of information deception. What can we do or what can an adversary do to corrupt the belief state of the decision maker?

Additionally, by approaching design from this perspective, we can develop solutions which enable the decision maker to employ their considerable problem solving skills to situations that may be novel, or were not recognized in the course of command activities. The approach is an attempt to augment a commanders skills rather than replacing them.

The basic technologies addressed in this research effort included knowledge representation, reasoning, and co-evolutionary game theory. Each technology support an aspect of the total solution. The representation of knowledge has a number of requirements that enable us to apply a number of technologies to produce the hybrid solution being sought. We need a technology that enables the construction of knowledge bases, that minimize transformations between conceptual reasoning and process reasoning systems, and augment a Peircean based abductive reasoning architecture. The most difficult of these requirements involves the transformation between conceptual and process reasoning. In conceptual reasoning we are attempting to identify some object or concept while in process reasoning we are having to recognize the concept but also the state and the allowable transitions in state

The reasoning engine is based on C.S. Peirce's model of scientific inquiry. This philosophical construct provides the foundation for how we as humans reason about situations new to us. This model consists of three reasoning capabilities; Abduction, deduction and induction. A crude way of looking at this suite of logic is abduction provides plausible hypotheses to explain an observation, deduction provides a basis for selecting from that set of hypotheses, and induction is the means to validate the hypothesis selected.

Co-evolutionary game theory provides the basis for assessing merits of a hypothetical solution against a suite of objectives. In this analysis we assume multiple objectives for each player in the game. What this means is, in a non-cooperative game, each player has the ability to evolve over time. The resultant Pareto optimal solution space identifies the best strategies any player can use to maximize their objectives. IN an IED domain that might mean the best design and deployment for an adversary, and the best detection and mitigation strategies for the blue side.

3. Formal Concept Analysis (FCA)

Formal concept analysis is a knowledge representation development effort initiated by Ganter & Wille based on ordered set theory. The mathematics of FCA lend themselves to lattice theory and the rich representation capabilities of that domain. FCA is based on the idea of a formal context, $\mathcal{K}_{FC}$, defined by a "triple" as the one in equation 1.

$$\mathcal{K}_{fc} \equiv (\mathcal{G}, \mathcal{M}, \mathcal{I}) \qquad \text{Eqn 1}$$

In this equation G and M are sets of objects and attributes respectively and I is a binary relation between the two sets. There is an operator defined, $(\cdot)'$ which aids in the definition of formal concepts from the formal context.

$$(A)' \equiv \{m \in \mathcal{M} \,|\, (g,m) \in I \;\forall\, g \in A\}$$
$$(B)' \equiv \{g \in \mathcal{G} \,|\, (g,m) \in I \;\forall\, m \in B\} \qquad \text{Eqn 2}$$

In this expression, the operator action on the object set A produces the set of attributes common to objects within that set. Likewise, application of the operator on the set of attributes B produces the set of objects which posses those attributes. The interesting application of this operator, which has very practical operational implications, is shown in equations 3.

$$A \subseteq ((A))'$$
$$(A)' = (((A)')')' \qquad \text{Eqn 3}$$

Operationally, this operator permits us to efficiently construct a working context based on data being processed to produce a complete object / attribute context. The first application of the operator identifies common attributes while the second application identifies objects possessing the attributes which were common to the original set of objects. The result of this operation can potentially be a larger object set than the original object set based on the formal context on which the operator is being applied. This is a very powerful tool for use in knowledge / data search.

The linkage to lattice theory provides avenues into a robust representation domain that can aid an analyst in developing an understanding of the collected data. The technologies use the "Begriff" of an identified context as the basis for the construction of that lattice. The Begriff, $\mathcal{B}$(G,M,I), is the ordered set of all concepts within a context. A concept is defined by the conditions in equation 4.

$$(A,B) \xrightarrow{fc} (\mathcal{G}, \mathcal{M}, I)$$
$$\Longleftrightarrow$$
$$\forall (A \subseteq \mathcal{G}, B \subseteq \mathcal{M}) \qquad \text{Eqn 4}$$
$$(A)' = B \;\&\; (B)' = A$$

The ordering of the concepts in $\mathcal{B}$(G,M,I) is defined in the next expression.

$$(A_1, B_1) \le (A_2, B_2)$$
$$\Longleftrightarrow \qquad \text{Eqn 5}$$
$$A_1 \subseteq A_2 \vee B_2 \subseteq B_1$$

An example of a lattice is given in from information developed by K. Wolff for his FCA tutorial. This example is a simple model capturing aspects of a knowledge base dealing with animals. In matrix representation the information is the following.

Animals	Preying	Flying	Bird	mammal
Lion	x			x
Finch		x	x	
Eagle	x	x	x	
Hare				x
Ostrich			x	
Bee		x		

Table 1. Matrix representation of an animal context.

The lattice representation of this information is shown in figure 2.

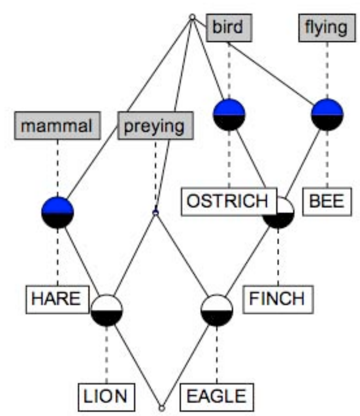

Figure 2. Lattice of animal domain.

The expansion capability of this technology is captured by the "Bee" entry in the matrix. The lattice prior to the addition of the information related to the bee consists of information in figure 2 with the upper right node (BEE) removed. Expanding a knowledge base, to include the bee, is a simple task in this technology. Likewise, the parsing of a lattice can be accomplished nearly as easily. What this does is give us the ability to structure the lattice at varying levels of knowledge abstraction and then when additional detailed information is of interest we can "zoom" into an object node to see the additional structure of the knowledge base under the selected node. This mechanical process adds to the potential understanding of knowledge and data being worked with.

The reality of the situation is that attributes are often defined by continuous real variables and / or may be probabilistic. Formal concept analysis deals with attributes with continuous variables by defining a special construct called a “many valued context”. They are defined in the next expression.

$$\mathcal{K}_{mv} \equiv (\mathcal{G}, \mathcal{M}, \mathcal{W}, \mathcal{I}) \qquad \text{Eqn 6}$$

As before, G is the set of objects, M is a set of attributes with values from the set W, defined by a ternary relational operator I. In this extension, the set of all values an attribute may assume is defined by the domain of that attribute.

$$dom(m) \equiv \{g \in \mathcal{G} \,|\, (g,m,w) \in \mathcal{I}\ w \in \mathcal{W}\} \qquad \text{Eqn 7}$$

To use many-valued contexts in formal concept analyses these attributes must go through a scaling process in order to generate a formal context that identifies the presence or absence of an attribute. Scaling can be considered as a construction of a special context that defines the relations of the many-valued attributes with new attribute sets and then ‘joining’ the original context and the new scale context. The new scaling context can be represent in equation 8.

$$S_m \equiv (\mathcal{G}_m, \mathcal{M}_m, \mathcal{I}_m) \qquad \text{Eqn 8}$$

$\boldsymbol{M}_m$ is a set of new attributes to represent the many-valued attribute in $\boldsymbol{G}$ and $\boldsymbol{I}_m$ is the binary relationship between the attribute sets. An example from Tam involves book prices.

	Price			>$0	>$25
Book A	$25.95		$25.95		x
Book B	$19.95		$19.95	x	
Book C	$74.95		$74.95		x

Table 2. Initial book price context and scaling context.

This results in a new context defined below.

	Price > $0	Price >$25
Book A		x
Book B	x	
Book C		x

Table 3. Scale context of book prices.

This process of scaling is important in Wolff’s extension of formal concept analysis into the temporal domain.

Dealing with uncertainty and probabilities of attribute associations has been treated in a more mechanistic fashion by the author. The Attribute sets carry a probability of association with an object into the lattice construction domain which is converted to a binary relationship based on a ‘threshold’ value identified by an analyst. This approach simplifies treatment of information uncertainty and lends itself to use by Finn’s instantiation of Mills first canon which requires the construction of exemplar lattices.

3.1. Mathematics of generating a Begriff.

Within the construct of this effort we have considered a context to represent a “related block” of information, e.g. an explosives data set, or a sports car data set. The idea of a related block of information becomes import in the section discussing fuzzy variable transformations. The Begriff is the set of all concepts of a context. The set of concepts can be defined by an application of the “prime” operator discussed in the previous sections. Determining the Begriff consists of applying the “prime” operator to each attribute and then to all combinations of attributes associated with the intent of the context.

$$\begin{gathered}
\mathcal{B} = \sum_n \left(\mathcal{F}^n\right) \\
with \\
\mathcal{F}^n = \sum_k \left(m_k\right) \quad for\ n = 1 \\
\mathcal{F}^n = \sum_j \left(m_j \sum_{k>j} \left(m_k\right)\right) \quad for\ n = 2 \\
\mathcal{F}^n = \sum_l \left(m_l \sum_{j>l} \left(m_j \sum_{k>j} \left(m_k\right)\right)\right) \quad for\ n = 3 \\
\vdots \\
n = \{1, 2, \cdots m\}
\end{gathered} \qquad \text{Eqn 9}$$

A procedural approach to defining a Begriff can be found in Davey & Priestley’s book. This approach relies on a process that uses a series of set intersections as the context is processed. The effect is the same while the equations above are a rigorous interpretation of the process described.

3.2. Likelihood estimates

Much of the effort has focused on laying the foundations for a rigorous theoretical suite of technologies. This translates into minimizing the number of heuristics employed in the formulations. In developing an engineering implementation of the theoretics this rigor is relaxed within the bounds defined by the theory. One engineering issue concerns false alarm / false negative types of issues associated with learning. In this situation we want to be able to tailor the bias of the resultant estimator. by defining the likelihood of a set of attributes being representative of a positive or negative example we can define thresholds of suitability.

The likelihoods are generated by the size of the concept at a particular node. This size is defined as the extent of the concept. normalizing to the total number of instances in the context provides an estimate of this likelihood.

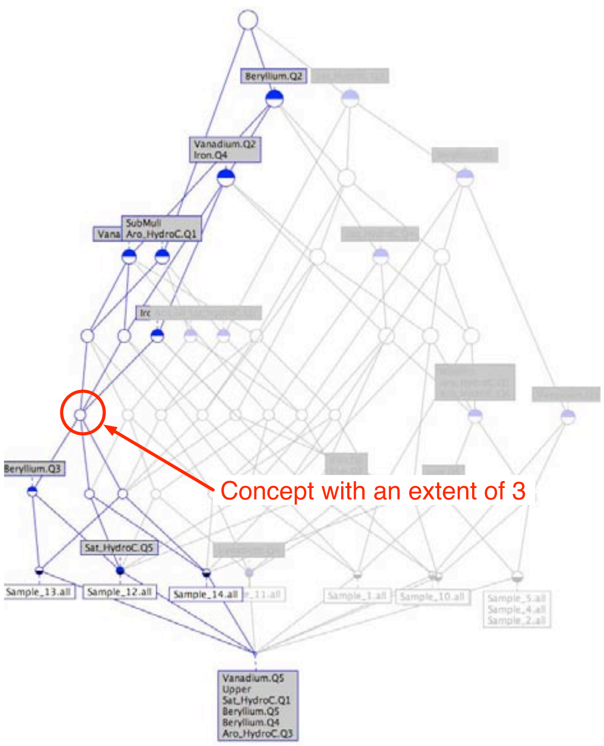

Figure 3. Example of a likelihood estimate.

$$L = S_{extent} / S_{context} \qquad \text{Eqn 10}$$

The S_{extent} represents the size of the concept and $S_{context}$ is the size of the context associated with the concept. While very simple it can effectively used with J.S. Mill's first canon which is a learning operator.

3.3. Fuzzy variable transformation

Formal concept analysis is based on a binary relationship operator between objects and attributes, either the attribute can be associated with the object or not. The problem is that in real situations many of the attributes may be real or even spectral in character. In order to transform real world information into a form amenable to FCA we use a "fuzzification" process based on fuzzy set theory. Within a context we assume that a real attribute posses a common interpretation. Temperature in a materials context should not be associated with temperature for a physical location. If all temperatures were lumped and fuzified significant biases would be introduced as well as introducing fidelity issues into the knowledge repository.

Identifying and isolating a real variable is the first step of the process. The range of that variable is determined and "padding" of 10% is added to the maximum and minimum values to ensure a degree of robustness to the context classification.

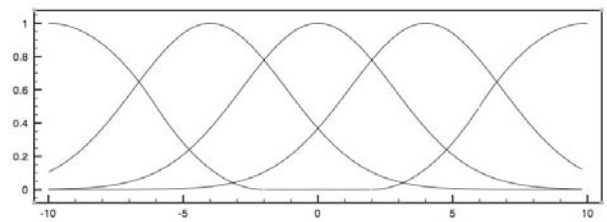

Figure 4. Fuzzification of a real variable over a range of -10 to 10.

The figure above shows a real fuzzification using 5 fuzzy levels. A variable value along the horizontal axis permits us to estimate the likelihood that the attribute belongs to a particular classification. In the implementation of the process we use a over laid structure which permits a greater combinatory representation of a variable. For example a variable value of '5', has non-zero membership in 3 quantiles of the fuzzified variable. This used in conjunction with the threshold variable gives an analyst a great deal of flexibility to discriminate information in a reasoning system.

4. Temporal Concept Analysis

Temporal concept analysis is an extension of FCA in which the evolutions of the system or object are considered in conjunction with the conceptual aspects of the object. The principle researchers in the area, Wolff and Neouchi, approach the problem by adding directed edges to the lattice to capture the evolutionary behaviors of the attributes. Wolff's efforts have resulted in a very formal representation of the temporal extensions of FCA while Neouchi has focused on the development / definition of sets of operators that focus on issues associated with temporal concepts.

Wolff has approached temporal concept analysis by scaling the time and event space and adding directed edges to the concept lattice of the context. The potential difficulty of this approach can be seen in the simple example in the next figure.

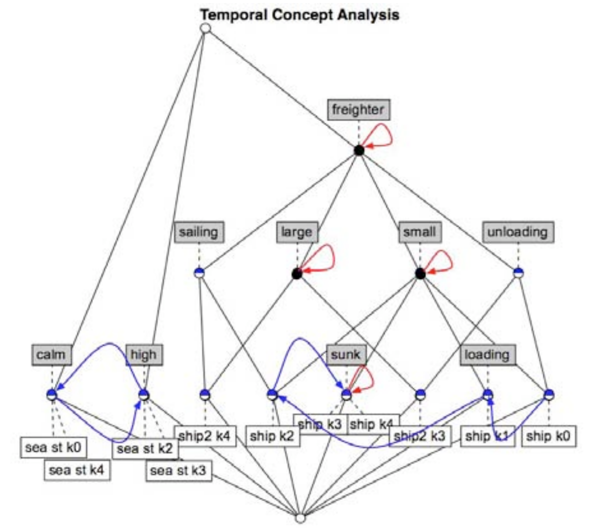

Figure 5. Example of lattice with directed edge overlay.

The blue vectors on the lattice in figure 5 indicate the temporal evolution of the objects in the formal context. The red vectors show persistent states of objects in that context. What I think becomes clear is the complexity of the display for even so simple an example. Complex information bases will rapidly overwhelm any advantages lattice representation bring to formal concept analyses.

A way around this complexity issue is to redefine how we think about systems / objects and the states of those systems. Traditionally, we view a system in a specific state as a unique object, so we are forced in a FCA paradigm to replicate an object as many times as we have states for it. If we instead view the system as being unique with sets of constant or time dependent attributes we can reduce the complexity of the lattice.

The paradigm we are working to develop is a 'zoomable' model in which we can zoom into an object to flesh out greater detail of the object at lower levels of conceptual abstraction. We can perform a similar function when approaching issues of systems state or the time dependent attributes. We can zoom into the temporal attribute and use the mathematics or technology that is better suited for the problem being solved. For example we can use FCA to move us into a conceptual neighborhood and focus on a temporal attribute and use Bayesian, Markov, or the temporally extended formal concept analysis to refine our understanding of a situation.

We might be able to see these possibilities in more detail by considering the information in figure 6. The notional example considers different temporal traces for the 4 attributes and a different set of attributes for two objects. We can see that taking a snapshot of these systems or objects at different points in time produces different collections of attributes for the objects. This can also change with different threshold levels. At point 'a', object 1 is characterized by attributes A while object 2 by attributes A and D. If D was not in the data set the correct hypothesis could not be identified. Using a process of temporal matching could refine the hypothesis since A is present in object 1 at all three states while it is only present at state 'a' in object 2.

Knowing the Markov transition matrix could aid in the proper identification of a temporally dependent hypothesis. Likewise temporal extensions of formal concept analysis could also be used to refine the selection mechanisms. The second approach may require additional computational overhead, but should be just as effective.

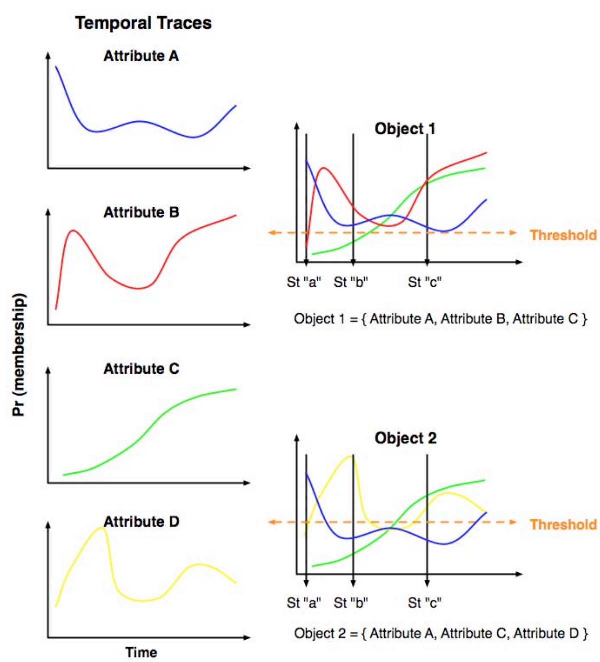

Figure 6. Temporal traces of four attributes and two objects with a mix of attributes.

4.1. Temporal logic.

Andre' Trudel discusses a concept of temporal logic in which information collected or understandings achieved affect not only future projects but also past experience. Effectively we may re-interpret a past event based on new information. This perspective ties into our understanding of belief states and needs to be addressed as belief is generated or updated during the course of analysis or experience.

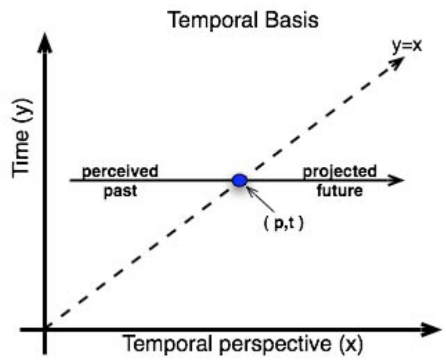

Figure 7. Model for temporal reasoning.

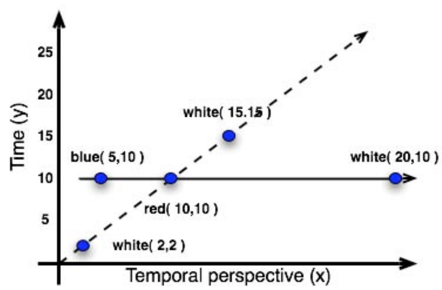

Figure 8. Temporal example.

The premise is that "the here & now" lies on the 45 deg. line defined by the x-y axis. This is indicated by the point (p,p). The perceived past lies on a horizontal line emanating from (p,p) but for x < p. similarly the expected future lies on the line but for x > p.

Figure 8 shows that at (2,2) a house is white; at (10,10) it is red but the agent thought it was blue at (5,5). This belief occurred at (10,10), thus the notation blue(5,10). At (15,15) the house was again observed to be white.

What the structure or the paradigm brings to the table is a way to think about how new information can impact past belief which in turn can impact projections or predictions.

5. Peircean Reasoning

Reasoning is the process we as humans use to solve problems or make decisions. We all use reasoning, some use sophisticated philosophies, others use ad hoc reasoning. The form taken is a function of our training and experience. Modal logic enters the equation in attempts to describe the flavors or nuances of reasoning we employ. The ultimate form of reasoning is the method of scientific inquiry which was defined by C.S. Peirce.

The three forms of reasoning in this paradigm consists of deduction, induction and abduction. Deductive reasoning is based on a structure that concludes if the premise of an argument is true the resultant must be true. Inductive reasoning operates on a principle that if "... I thrown a ball in the air it fell to the ground every time..." I believe that the next time I throw the ball in the air it will fall to the ground. Abduction is the more complex form of reasoning, in this case we develop hypotheses based on knowledge we possess to explain a new set of observations. Peircean reasoning is a hybrid form that integrates these three foundational forms of reasoning into his method of scientific inquiry.

The reasoning engine implemented in this effort is based on C.S. Peirce's model of scientific inquiry. This philosophical construct provides the foundation for how we as humans reason about situations new to us. This model consists of three reasoning capabilities; Abduction, deduction and induction. The logic associated with these forms of reasoning are captured in figure 9.

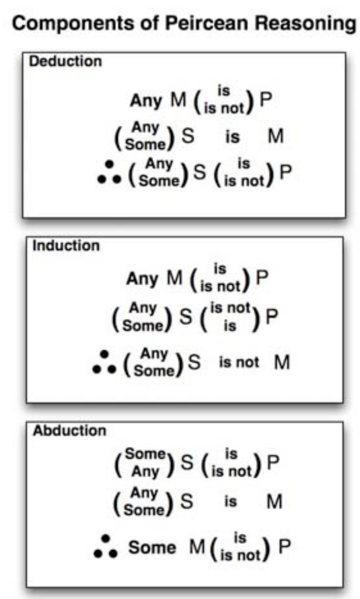

Figure 9. Formal representation of Peircean reasoning.

A crude way of looking at this suite of logic is abduction provides plausible hypotheses to explain an observation, deduction provides a basis for selecting from that set of hypotheses, and induction is the means to validate the hypothesis selected. Induction can be viewed as a statistical collection of data that confirms or supports the hypothesis. This statistical validation must be tempered by maxims such as "severe" testing as defined by Mayo. A second nuance of this problem is the frequentist perspective that needs to be tempered by Bayesian statistics for many of the problem domains this solution is being proposed to address.

Not addressed in this effort is analogical reasoning which is a form of abductive reasoning. The classic example of analogical reasoning is the Bohr atom example. electron's revolve around the nucleus like planets revolve around the sun. Therefore, the forces in an atom can be modeled using an inverse-square law. This form of hypothesis generation examines the detail of phenomena and looks for similarities at these levels to draw higher level hypotheses.

5.1. Modal Logics

Modal logic deals with possibility and plausibility in reasoning processes. Some of the more familiar forms of modal logic include Kripke, Deontic, Temporal, and Doxastic logic. Kripke logic seems to form a basis set upon which are added logic operators for a specific domain or issue associated with reasoning about the truth of some argument. Temporal logic is possibly the easiest to understand which deals with assertions functionally dependent in time. E,g, "it will always be that" or "it was always that" are 2 operators from temporal logic.

$(1) \models K(\alpha \to \beta) \to (K\alpha \to K\beta)$
$(2) \models B(\alpha \to \beta) \to (B\alpha \to B\beta)$
$(3) \models K\alpha \to \alpha$
$(4) \models K\alpha \to B\alpha$
$(5)\ \mathit{if} \models \alpha, \mathit{then} \models K\alpha$
$(6)\ \mathit{if} \models \alpha, \mathit{then} \models B\alpha$

$(7)\ \neg B\perp$ — *Consistency*
$(8)\ BB\alpha \to B\alpha$ — *Veridicality of Pos Introspection*
$(9)\ \neg B\perp \to (B\neg B\alpha \to \neg B\alpha)$ — *Veridicality of Neg Introspection*
$(10)\ B\alpha \to BB\alpha$ — *Positive Introspection*
$(11)\ \neg B\alpha \to B\neg B\alpha$ — *Negative Introspection*

Definitions:
$K \sim$ *Knowledge*
$B \sim$ *Belief*
$\models \sim$ *"it is logically valid"*
$\perp \sim$ *a logical contridiction*
$\alpha, \beta \sim$ *represent blocks of info / knowledge*

Figure 10. Logic dealing with the veridicality of knowledge.

A very interesting implementation of a modal logic is from Lindstroem & Rabinowicz which deals with knowledge and belief. As we begin to formulate a total solution we find situations requiring knowledge update(epistemic logic), belief revision(doxastic logic) temporal logic and others to formulate a complete, theoretically founded solution.

5.2. Knowledge Operators

The operators being defined or designed for this reasoning construct are based on the five canons of John Stuart Mill. Initial work by Burch and Finn have focused on the first canon and involved significant effort at validating these canons in a much larger philosophical and logic context. In this effort we are taking a more Peircean, pragmatic approach to selecting and implementing the operators. The five canons consist of those identified in the next list.

J.S. Mills Canons

- Method of Agreement
- Method of Differences
- Indirect Method
- Method of Residues
- Method of Concomitant Variables

The first canon: *If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon.*

The second canon: *If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.*

The third canon: *If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of the circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or cause, or a necessary part of the cause, of the phenomenon.*

The fourth canon: *Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.*"

The fifth canon: *Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*

The descriptions of the canons come directly from Mill's *System of Logic*, and will form the basis for the knowledge operators in the system. Only the first 2 canons have been implemented in the coded algorithms.

5.2.1.Implementation of JSM-1. The first canon, the method of agreement, addresses issues of learning. The Finn implementation involves the construction of 3 contexts, a positive, a negative and an unknown context. The positive context captures examples which are representations of a goal attribute, the negative context provides counter examples and the unknown or neutral context represents a set of instances to be classified. In his formulation Finn identifies two types of attributes, structural and gaol attributes. Structural attributes are those describing an instance. The goal attribute is an attribute which describes a common characteristic. An example developed involves the classification of the field from which crude oil was pumped. In the example the goal attribute was "field" which had examples from 3 production fields, "Upper", "Wilhelm", and "SubMuli".

Our implementation is modified for a number of reasons, first, to eliminate a heuristic in the Finn formulation and second, follow a paradigm of a knowledge base. The knowledge base contains classification contexts and knowledge derived from learning contexts. This last adjustment uses the first canon to construct a context in which conditions(attributes) are identified that are characteristic of the goal condition. Figure 3 in the section on likelihood estimation is a portion of the learning lattice used to construct the classification knowledge concerning oil field characterization.

$$\begin{aligned} C_{lc} &\Rightarrow \mathcal{B}^+, \mathcal{B}^- \\ L_{goal} &= \mathcal{B}^+ - \mathcal{B}^+ \cap \mathcal{B}^- \end{aligned} \qquad \text{Eqn 11}$$

A learning context is converted to a positive and negative begriff which is subtracted from the positive begriff producing an incomplete goal lattice. The set nature of the begriff requires that the subtraction operation be defined as in equation 11. The resultant goal lattice is then converted to a classification context which when displayed in a lattice looks like the next figure.

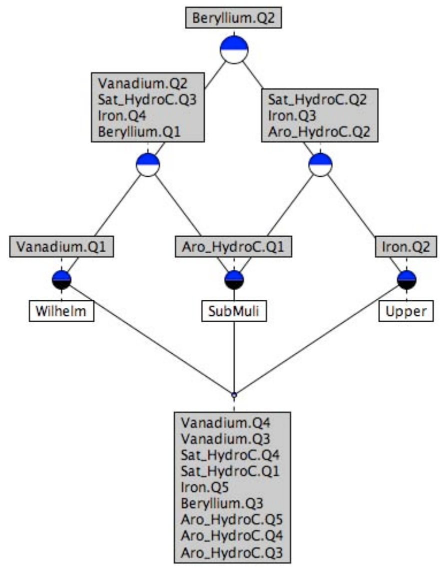

Figure 11. Learned classification context.

In this figure the 3 oil production fields are delineated in the first row of the lattice. Beryllium.Q2 and Iron.Q3 would lead to a two hypotheses, the oil came from the SubMuli or Upper fields. A deductive screen would indicate that a test for Aro_HydroC.Q1 would indicate the field was SubMuli while the lack of that attribute would mean the sample came from the Upper field. This lattice is the result of 54 samples from the 3 fields.

The engineering aspects come into play through a threshold used in conjunction with the likelihood estimates described earlier. Imposing a likelihood or greater than 0.2 on concepts in the positive begriff means that a single example may not be sufficient to constitute a positive example. This process reduces the probability of predicting unknown instances but reduces false positives. Similarly, by imposing a threshold on the negative begriff results in reducing false negatives in a prediction problem.

5.2.2. Implementation of JSM-2. The second canon, the method of differences, can be characterized as a causal reasoning operator. In this case we again split evidential context into positive and negative begriff's. The first step is to identify common attributes for the positive evidence. Once we have created that positive common lattice we subtract concepts of the negative begriff from this common lattice. mathematically the operations are described in the next equation.

$$\begin{aligned} C_{ev} &\Rightarrow \mathcal{B}^{+}, \mathcal{B}^{-} \\ (\mathcal{L}^{+})_{common} &= \mathcal{B}^{+}_{1}\left(\sum_{k}\left(\cap\, \mathcal{B}^{+}_{k}\right)\right) \\ L_{cause} &= (\mathcal{L}^{+})_{common} - (\mathcal{L}^{+})_{common} \cap \mathcal{B}^{-} \end{aligned} \qquad \text{Eqn 12}$$

5.2.3. Implementation of JSM-3. Interestingly, the third cannon is a very simple variation of the second. The difference is the intersection term in the 3rd expression of equation 12 is "zero" based on the description above. The result is that the algorithm for JSM-2 will also support JSM-3.

6. Co-Evolutionary Game Theory

Game theory as defined by L. Samuelson is the study of interactive decision making. In general, the essentials of game theory include games played either cooperatively or non-cooperatively with 2 or more players. The game can be played once or repeated by rational players with known utility functions(goals/objectives). In single play Nash equilibrium is the ultimate stability point while in evolutionary game theory where all players evolve the stability point is defined by the idea of evolutionary stable strategy(ESS) in which the ESS does not necessarily correspond to a Nash equilibrium.

Classic game theory is played one time with a payoff matrix defining the results of the game based on the strategies. In pure game theory Nash equilibrium is the condition that results from a mixed strategy and constitutes the best possible result of the game in which the players are rational players.

Evolutionary game theory modifies the game by playing repeated games, again in a non-cooperative environment. In this approach, each time the game is played the game participants are drawn from a population of players each having the same or different strategies of play. Under the rules of this game a process is defined for modifying the population of players. The operators defining the modification can be designed based on the objectives of the game. The other significant difference of this game theoretic approach involves the fact that there is no guarantee that the solution will evolve to a Nash equilibrium. In the case of evolutionary game theory the solutions evolve to the evolutionary stable strategy or ESS.

Finally, in a co-evolutionary game theoretic environment we are removing all restrictions on the nature of the game. In this case it could be thought of as a game in which the rules are changing as well as the playing pieces. The strategies, rather than being defined by a population of potential players, are being defined by reasoning entities integral to the game. The implications of this approach are not yet clear, but we expect to see behavior similar to the evolutionary game theoretic in which we evolve to an ESS.

As problem solvers we may apply a manifestation of game theory as part of our reasoning process. When we speculate on possible solutions, internally we are assessing these solutions using a heuristic or model that translates initial conditions into some sort of effect. In the military decision making process (MDMP) the war gaming done in support of military planning is or can be an external manifestation of that evolutionary game in which we assess the impact of decision being made.

Research in the area of co-evolutionary game theory has focused on cooperative games in which each side uses a single weighted fitness function for each player in the game. The game is played to some definition of optimization using a Pareto approach. The problem in this approach involves the need for non-cooperative game play and a need to address multiple objective for all the players. No one, except American voters, are single objective individuals. In the complex environments of insurgencies and terrorism the players possess many levels and classes of objective. When one objective is blocked, a shift occurs to maximize along another objective dimension.

Performing a co-evolutionary Pareto optimization against a spectrum of objectives for each player provides a far more robust and dynamic solution that can be used in the field. Moving around the Pareto surface in response to changes in preference by one or more players ensures any perceived advantages can be mitigated by the other players.

6.1. Pareto Optimization

Pareto optimization is similar to evolutionary optimization in that a massive directed search of the solution space is conducted in an effort to find the "best" solution. The fitness function in a Pareto optimization involves dominance criteria over the objective dimensions of the problem. If we consider a multi-objective function, Pareto optimization can be defined by the following expression for a minimization problem.

$$F(f(\overline{x})) \equiv \left[f_1(\overline{x^*}), f_2(\overline{x^*}), \cdots, f_m(\overline{x^*}) \right]^T$$
$$\overline{x^*} \sim represents\ a\ solution \qquad \text{Eqn 13}$$
$$f_k(\overline{x}) \leq f_k(\overline{x^*}) \quad for\ k = 1, 2, \cdots, m$$

In this expression F() is the vector of objectives and a non-dominated solution satisfies the condition in the 3rd expression of equation 13. These non-dominated solutions fall on the line defined by the blue bullets.

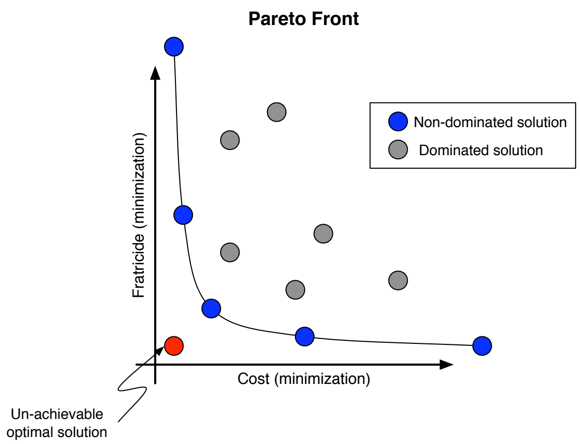

Figure 12. Pareto optimal curve.

7. Data / Information Fusion

Considering data / information fusion as the basis for an information architecture, we begin to see the impacts on the other elements of the systems design. In Hawkins book "On Intelligence", we see a model for the neocortex which ideally suites the needs of an information fusion paradigm that supports the essential elements of a reasoning based approach to fusion. In his model he articulates a layered system in which different levels of abstraction are realized at each of the six layers. Figure 14. takes a little liberty in representing the model Hawkins proposed and adds a technical solution to the representation of the layers of the neocortex. The points to capture from this construct is the comprehensive feedback loops between layers of the neocortex and the links to different sensors, like auditory, visual, smell, etc. The feedback loops activate an "expectation mechanism, when performing a similar function daily we expect things to be the same as the day before. Opening the door to your office, we expect to find a round smooth knob which must be turned. When that knob was changed overnight and we now discover a lever, we stop and have to adjust or discover a method for entering that door. We have effectively shifted from an inductive-deductive pattern matching system to an abductive based system.

In a similar way, the expectation crosses sensor boundaries such that not only do we expect a certain feel to the door knob but we expect to hear that familiar squeak, also a silver color and the knob to be at room temperature. When any of these conditions have changed we shift to an abductive problem solving paradigm.

In the model presented in figure 14, we have represented each layer as an ART(adaptive resonance theory) neural network. The reason for selecting this initial technology is because of the classification capability of that design. We are looking for a technology that correlates attributes with instances, or an object with state variables. An object can in turn be a member of a higher level set of attributes which define a more complex abstraction. This abstraction concept becomes important for the enablement of high level reasoning and can be supported by a knowledge representation technology based on formal concept analysis.

The deliberative aspects of a fusion system, when the Hawkins net does not find a definitive solution, is provided in the next figure which begins to show the integration of modal logic into the solution.

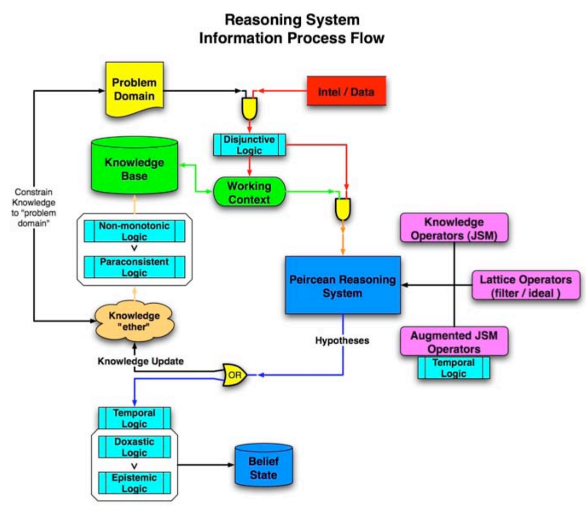

Figure 13. Reasoning engine for use in fusion systems.

In this model we see instances of modal logic delineated that is used to process the information that is used by the reasoning engine. This engine may also employ modal logics of a form appropriate for the problems being addressed.

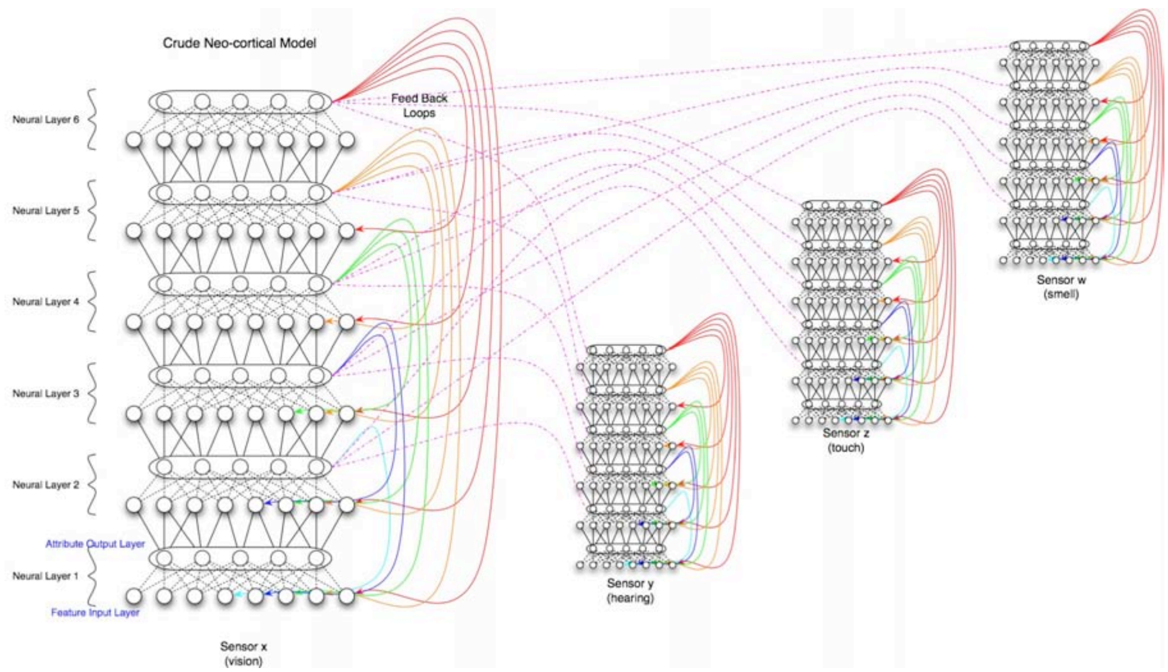

Figure 14. Hawkins net based on ART NNs.

8. Predictive Decision Aids

What has been addressed up to this point are the core technologies which form the foundations of a fusion system. As indicated this fusion system needs to include a reasoning component to begin to address the complex issues of data and information fusion. Anything short of that design is wasting time and resources. The last topic I want to touch on involves predictive decision aids. This last integration step involves integrating the fusion/reasoning engine into the co-evolutionary game engine.

The objective is to create an environment that postulates solutions to some mission objective and tests them in a competitive scenario. The solution requires a simulation capability as well as the Pareto optimization mechanisms. A system architecture is provided in the next figure.

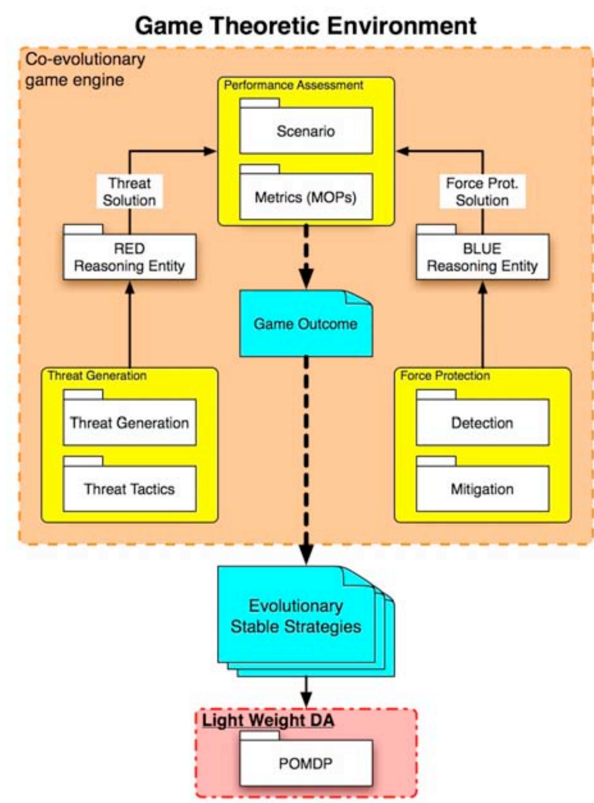

Figure 15. Simple model of the co-evolutionary game engine.

What is being depicted is a co-evolutionary game environment in which all players evolve with experience subject to the scenario and their objectives. The normal "bag of strategies" is replaced by reasoning engines that formulate solutions based on their knowledge. The game searches for an evolutionary stable strategy which is an n-dimensional Pareto surface. That information is transfered to a light weight decision aid for use by a force commander.

8.1. Belief State (Cache)

The belief cache can be viewed as the tagged collection of validated hypotheses generated by the reasoning system. This cache contains the understanding up to the current point in time, of data being collected and assessed. In a combat environment this belief cache can be interpreted as "situational awareness". The structure of this cache is defined in the next equation.

$$Bk_j = \left\langle \begin{array}{c} t_j, Active_j \\ \{h_{j,0}, d_1, \ldots d_n, d_1^v, \ldots d_m^v\} \\ \{h_{j,k}, d_1, \ldots d_n, d_1^v, \ldots d_m^v, h_{j,k-1}, \ldots h_{j,k-t}\} \\ \{h_{j,r}, d_1, \ldots d_n, d_1^v, \ldots d_m^v, h_{j,r-1}, \ldots h_{j,r-s}\} \end{array} \right\rangle$$

These belief kernels consist of a time tag, t_j an activation flag, *Acitve*$_j$, a hypothesis, h_j, data collected that results in the hypothesis, d_n and data collected to validate the hypothesis, d^v_m. The next two notional inclusions consist of hypotheses from higher levels of abstraction that may depend on hypotheses generated at sets of lower abstraction. This construct is needed to trace the impact of changes or updates to information at lower levels of abstraction.

9. Summation

What we have identified in this short note is a suite of technologies that together define a solution to fusion which captures a reasoning model that supports fusion. It is this approach that is needed if we are to capture the human capability of performing fusion which has at its core a reasoning function. The solution we are working towards is a 70-80 percent solution, to demonstrate the synergistic functioning of the major technologies we have identified as integral to that solution.

Significant additional work needs to be performed to ensure the optimal identification of the modal logics required by the solution. There may be a better mix, or alternatives that have not been realized. Logic has implications on the information security, on its timeliness, on its validity, and its quality. Modal logics also aid in the management of knowledge and the belief. The effort here has only scratched the surface, but the importance of this integration cannot be missed or ignored.

The knowledge representation technology of formal concept analysis is in my opinion, the best suited to support logic, reasoning, and the neocortical architecture identified as the real time fusion engine. It also seems to support the two major forms of reasoning that we need in decision aid problems were we need to be able to perform concept reasoning as well as process or temporal reasoning.

Finally, a fusion solution requires a core reasoning capability. When the inductive – deductive functioning of the system cannot identify a situation you need to be able to switch into an abductive hypothesis generating function in the effort to find a solution to this new situation. Hawkins neocortical model provides a fast running induction-deduction engine, that additionally supports Peircean reasoning, is a natural for multi-sensor fusion, and the feedback mechanisms are a very powerful approach for prediction / expectation functionality.

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